A quantitative approach to carbon price risk modeling

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Abstract

The climate change discussion in the framework of the Kyoto protocol has clearly emphasized the need for reliable methods to value projects targeted on reduction of greenhouse gases. Moreover, the recent carbon price development in the mandatory EU Emission Trading Scheme exhibits the insisting importance of accurate risk management when business is exposed to greenhouse gas emissions. In this work we suggest a model for price formation of carbon emission rights.

Key words: commodity options, environmental risk, energy economics, carbon allowances, Kyoto protocol

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1 Introduction

The world's changing climate and actions designed to curtail the man-made effects on ecosystems continue to challenge policy makers. Here, one of the urgent themes is to establish regulations which provide a certain amount of flexibility, enabling agents to apply commitments for GHG reduction best suited to their circumstances.

The idea to use market-based asset trading systems in order to achieve environmental targets goes back to [4] and [10]. The authors of these contributions envisaged a market for the public good *environment* introduced by tradable emission credits. Using a static model for a perfect market with pollution certificates in [10], it is shown that there exists a minimum cost equilibrium for companies facing a given environmental target. The conceptual basis for dynamic permit trading is, among others, addressed in [3], [16], [13], [8], [14] and [15]. This line of work is related to our approach since it treats the aspects of banking, borrowing, and the problems of inter-temporal strategy optimization in a multi-period setting. The recent work [15] suggests also a continuous-time model for carbon price formation. Beyond these themes, there exists a vast work on several related topics, for instance on equilibrium [2], empirical evidence from already existing markets [7], initial allocations, technology relations [1], uncertainty and risk [5], [9], [17] and game-theoretic questions addressed in [11], [6].

Below, we give a short overview how different international, national and corporate policy frameworks are used to reduce GHG pollution in a flexible, cost effective manner by the introduction of marketable carbon emission credits. Thereby, we focus on the carbon emission market EU ETS designed by the European Community as an instrument to meet targets under the Kyoto protocol.

In 1997 governments adopted the Kyoto protocol that broke new ground with its mandatory constraints on reduction of GHG emissions. On the core of this agreement is a target to reduce GHG emissions for developed countries and economies in transition (the so-called Annex I members). These countries are set a legally binding cap for GHG emission to 5.2 % below their 1990 level. This reduction is to be attained in sum over all Annex I members in average over five years 2008 – 2012. The concrete implementation is as follows: Each Annex I member is assigned a carbon dioxide credit which equals to (5 years) $\times$ (country's emission in 1990) $\times$ (1-0.052). This credit is measured in the so-called Assigned Amount units (AAUs), corresponding to one ton of carbon dioxide.

Moreover, the Kyoto protocol contains three mechanisms: International Emission Trading (IET), Joint implementation (JI), and Clean Development Mechanism (CDM) that are designed to support Annex I members to meet their targets by purchasing emission credits from other parties, in particular

- for the IET mechanism, these credits are AAUs from other Annex I members
- for the JI mechanism, credits are the so-called ERUs (Emission Reduction Units) and RMUs (Specific Removable Units) which are obtained from projects within the Annex I area
- for the CDM mechanism, that are the so-called CERs (Certified Emission Reductions), obtained from GHG reduction within non-Annex I countries.

On this account, the need for trading of diverse emission allowances (AAU, ERU, RMU, CER) is obvious. Currently, futures and spot markets for these assets are being developed. Moreover, there is a number of regional and national emission reduction projects, where similarly to the above setting, agents trade diverse emission certificates.

A remarkable example of such a project is the European initiative *EU Emission Trading Scheme* (EU ETS), launched by the Directive 2003/87/EC of the European Parliament and the Council of 13th of October 2003. This scheme is intended to ensure reduction of GHG emissions from large industrial sources within the European Union as a contribution towards EU's targets under the Kyoto Protocol. EU ETS imposes mandatory participation of more than 12,000 installations. These are power plants and industrial users responsible for approximately 45 % of the entire EU carbon emission. Among these installations, carbon emission allowances (EUAs) are yearly allocated by the responsible governments, according to the corresponding National Allocation Plan (NAP). Installations have to cover their emissions by allowances year by year. To do so, they are able to procure credits CERs (from 2008 onwards also ERUs) and to trade in EUAs. The precise regulation is as follows: There are two periods 2005–2007 and 2008-20012. Within each period, allowances are valid for compliance against emissions regardless of the year they are allocated. At the end of each period, a penalty of is to be paid for each ton of emitted carbon dioxide which is not covered by allowances. The size of the potential payment is considerable: At the end of the first period, the penalty is amounted at 40 EURO plus one AAU from the second period, whereas at the end of the second period agents have to pay 100 EURO per ton of carbon dioxide in the case of non-compliance.

EU ETS has earned the reputation of a leader in fighting the climate change, especially as there is a global reach that facilitates sustainable development, in particular through the settlement of emission rights trading. Several exchanges in Europe are now committed to the trading of EUAs. The products listed there are spot and forward contracts with physical delivery. Exchange trading mitigates counterparty risk and is able to boost liquidity. The development of carbon markets towards increasing standardization raises also questions of pricing diverse financial contracts related to carbon. The present work is devoted to this problem.

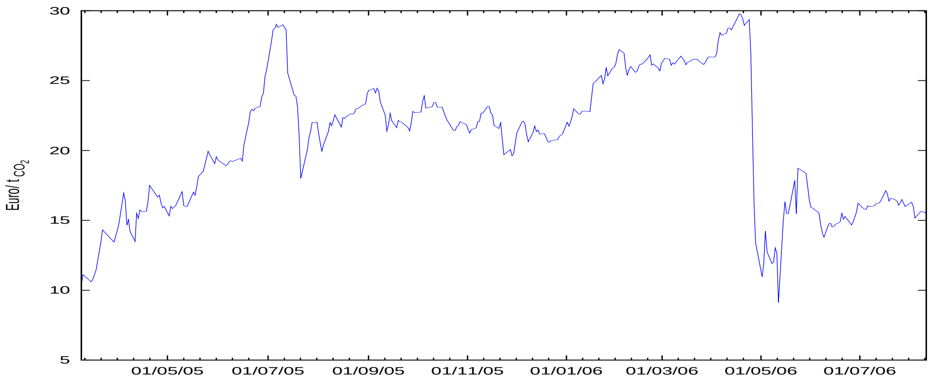

Figure 1: EUA spot price, listed at the European Energy Exchange EEX. The recent price drop occurred while carbon emission data became public showing that the overall market position is long

Let us explain why we feel that the valuation of carbon derivatives is important, although not many options are issued yet. The point here is that any GHG project, by its nature, yields a payoff depending on (even expressed in) market prices of carbon allowances at future times when the project returns GHG reduction. Thus, we have to consider any purposed investment in CDM– or JI–undertaking as a long position in appropriate option written on upcoming spot prices for carbon allowances. Moreover, a related aspect, referred to as *carbon finance* has recently attracted the attention of international investors. In this concept, investment decisions yielding carbon emission savings in form of energy consumption reduction (or its production from renewable/low-emitting sources) are examined with respect to all potentially generated "carbon assets" . The revenue from these assets can be used to (partially) pay back the investments. Needless to say, that the success of projects including a carbon finance component is determined by the correct valuation of their returns, and these cash-flows are equivalent to derivatives written on future carbon prices.

In a cap and trade system (like EU ETS) the allowance prices are determined by the existing abatement strategies, in particular by their costs and their flexibility. In the case of EU ETS, we have to distinguish between two types of abatement policies, referred to as *long term* and *short term* measures in the sequel. As the name reveals, the main difference is the time horizon needed to return the corresponding carbon emission savings. Furthermore, the long-term measures require high investments (fixed costs) which are, in general irreversible, whereas short-term measures cause merely costs which are proportional to the saved amount of carbon (variable costs). Examples of long-term measures are optimization/substitution of high polluting production units, installation of scrubbers, investment in CDM-and JI- projects. On the contrary, short term abatement measures yield emission savings within days, typically by replacing fuels or skipping/re-scheduling the production. The main short term abatement potential stems from the electricity sector, where the production is switched from hard coal to gas. On this account, commodity price models (in particular, fuel price models) form an intrinsic part of carbon price description. Consequently, we attempt to find out how the emission allowances price evolution is quantitatively related to the fuel price development and emission influencing factors, e.g. weather, plant outages. Both, long and short term abatements influence allowance prices in related but conceptually different ways, which are incorporated in the following model.

2 Mathematical model

In this approach, a stylized cap and trade system is considered, where market participants trade allowances and apply long and short term abatement policies to fulfill the compliance.

We proceed from the realization that the carbon price development reflects the private economic interests of installations, concerned by emission regulations in a cap and trade framework. Thus, the main aspect in our modeling is to face the individual strategy optimization of single market participants, exposed to carbon price risk. We consider personal incentives of stylized agents who possess the flexibility of short term emission reduction, which is exercised whenever emission allowance prices indicate that this is reasonable. On this account, commodity price models (in particular, fuel price models) form an intrinsic part of carbon price description. Consequently, we attempt to find out how the emission allowances price evolution is quantitatively related to the fuel price development.

We consider $N \in \mathbb{N}$ market participants trading carbon allowances at the discrete times $\{0, 1, \dots, T\} \subset \mathbb{N}$. At these times, they also produce electricity from fossil fuel. The entire time horizon corresponds to one compliance period, that is, at maturity T all agents have to cover their carbon emissions by allowances or, on the contrary, to pay penalties. Let us introduce the model ingredients.

We agree to describe all prices and strategies by appropriate adapted stochastic processes on a filtered probability space $(\Omega, \mathcal{F}, P, (\mathcal{F}_t)_{t=0}^T)$.

Write $A = (A_t)_{t=0}^T$ for the futures price with delivery date T of carbon allowances and assume that it follows a positive–valued stochastic process. Note that due to the definition of the futures price, A_T equals to the spot price for carbon allowances at the final time T. A futures trading strategy of the agent i is given by the process $\theta^i = (\theta^i_t)_{t=0}^{T-1}$, where θ^i_t for $t = 0, \dots, T-1$ stands for the number of futures contracts held by the producer $i = 1, \dots, N$. The wealth $(V_t^{\theta^i, A})_{t=0}^T$ of such a strategy follows the recursion

$$V_{t+1}^{\theta^i,A} = V_t^{\theta^i,A} + \theta_t^i(A_{t+1} - A_t) \quad t = 0, \dots, T-1, \quad V_0^{\theta^i,A} = 0 \tag{1}$$

reflecting the cash settlement of the future at any time $t = 0, \dots, T$. At maturity T, each producer i faces the difference between its actually emitted carbon and that allocated at the beginning. We model this quantity by an $\mathcal{F}_T$–measurable random variable Γ^i. Note that we allow for both, negative and positive realizations of Γ^i, occurring if the credit exceeds or drops below the actually realized demand . To fulfill the compliance at the end of the period, each agent adjusts the number of credits depending on the actually realized demand. We describe this action by the number of allowances θ_T^i which the agent purchases/procures at T giving the final emission balance $\Gamma^i - \theta_T^i$. In the case $\Gamma^i - \theta_T^i \geq 0$ the emissions top the amount of allowances owned at the end, a penalty of $\pi \in]0, \infty[$ EURO is to pay for each ton which is not covered. Thus, the final wealth from trading allowances equals to

$$V_T^{\theta^i,A} - \theta_T^i A_T - \pi(\Gamma^i - \theta_T^i)^+. \tag{2}$$

Remark The individual allowance demand Γ^i describes the compliance gap of the agent i at maturity T. Here we can agree that the long-term abatement measures are included in the sense that the emitted carbon is already adjusted by the estimated reduction from the proposed long-term measures. Thus, in this contribution, we do not explicitly consider the impact of the long-term emission reduction policies since their optimization follows its own logic and the outcome is well-foreseen.

Further, we model short term abatement strategies. Let us assume that each agent $i = 1, \dots, N$ observes its *own short term abatement* price $(E_t^i)_{t=0}^{T-1}$ which is supposed to follow an adapted stochastic process (note that fuel switch price is not necessarily positive). To give the reader an intuition, we illustrate the price of fuel switching from coal to gas in electricity generation, *which is the main short term abatement measure in the EU ETS.*

Fuel switching Consider an agent i switching from a hard coal plant to a cleaner CCGT (Combined Cycle Gas Turbine) plant. In this case, the agent's technology possesses specific emissions for gas

$$e_g^i = 0.202 \frac{\mathrm{t_{CO_2}}}{\mathrm{MWh_{therm}}} \cdot \frac{1}{0.52} \frac{\mathrm{MWh_{therm}}}{\mathrm{MWh_{el}}} = 0.388 \frac{\mathrm{t_{CO_2}}}{\mathrm{MWh_{el}}} \tag{3}$$

and coal

$$e_c^i = 0.341 \frac{\mathrm{t_{CO_2}}}{\mathrm{MWh_{therm}}} \cdot \frac{1}{0.38} \frac{\mathrm{MWh_{therm}}}{\mathrm{MWh_{el}}} = 0.897 \frac{\mathrm{t_{CO_2}}}{\mathrm{MWh_{el}}} \tag{4}$$

measured in ton of emitted carbon for the generation of one MWh of electricity. (Here, $\mathrm{t_{CO_2}}$ and $\mathrm{MWh_{therm}}$, $\mathrm{MWh_{el}}$ denote the ton of carbon dioxide and the Mega Watt of thermic, electrical power respectively. The CO_2 emission factors are default values provided by the Intergovernmental Panel on Climate Change (IPCC)). The switch of production technology at time t yields per MWh of electricity a reduction of $e^i_c - e^i_g = 0.509$ ton carbon dioxide. At the same time, this fuel switch causes costs of $h^i_g G^i_t - h^i_c C^i_t$ EURO per MWh, where G^i_t, C^i_t are gas and coal spot prices for the agent i at time t (expressed in EURO per $\text{MWh}_{\text{therm}}$ and in EURO per ton respectively). The coefficients

$$\begin{aligned} h^i_g &= \frac{1}{0.52}\frac{\text{MWh}_{\text{therm}}}{\text{MWh}_{\text{el}}} = 1.92\frac{\text{MWh}_{\text{therm}}}{\text{MWh}_{\text{el}}} \\ h^i_c &= \frac{1}{6.961}\frac{\text{t}_{\text{coal}}}{\text{MWh}_{\text{therm}}}\frac{1}{0.38}\frac{\text{MWh}_{\text{therm}}}{\text{MWh}_{\text{el}}} = 0.378\frac{\text{t}_{\text{coal}}}{\text{MWh}_{\text{el}}} \end{aligned}$$

are specific heating rates, expressing how much fuel is consumed for generation of one MWh of electricity. (Here we have assumed that the amount of coal is measured in ton, whereas the gas amount is expressed in Mega Watt hours of thermal power since gas prices we use are given in EURO/$\text{MWh}_{\text{therm}}$. The calculation of h^i_g is based on the reference value of 6000kcal/kg reported in McCloskey's NWE Steam Coal Marker.) With these quantities, we have the fuel switching price

$$E^i_t = \frac{h^i_g G^i_t - h^i_c C^i_t}{e^i_g - e^i_c} \qquad \text{for all } t = 0, \dots, T-1 \tag{5}$$

measured in EURO per ton of carbon dioxide. Based on a given time series for coal and gas spot prices, the formula (5) yields the corresponding fuel switching price process.

In commodity business, companies which are exposed to risks from several input commodities hedge themselves by an appropriate futures portfolio, thus price correlations of diverse commodities become essential. In particular, the European energy business is concerned about correlations of EUA and fuel prices. However as can be seen in Figure 2 their interdepence is not obvious. A study based on our model could shed light on this important aspect.

Let us turn back to the modeling of carbon price formation. We suppose that each producer i possesses a technology which at any time $t = 0, \dots, T-1$ allows a reduction ξ^i_t of at most $\lambda^i \in [0, \infty[$ ton of carbon emitted within the period $[t, t+1]$ by fuel-switching. The fuel switching policy $\xi^i = (\xi^i_t)_{t=0}^{T-1}$ yields expenses which are modeled by cash payment of the amount

$$\sum_{t=0}^{T-1} \xi^i_t \frac{E^i_t}{p_t(T)} = \sum_{t=0}^{T-1} \xi^i_t \mathcal{E}^i_t \tag{6}$$

at maturity T. Here, to incorporate interest effects, we have related the fuel switch spot price E_t to the price $p_t(T)$ of the zero bond maturing at T by

$$\mathcal{E}^i_t := E^i_t / p_t(T) \qquad t = 0, \dots, T.$$

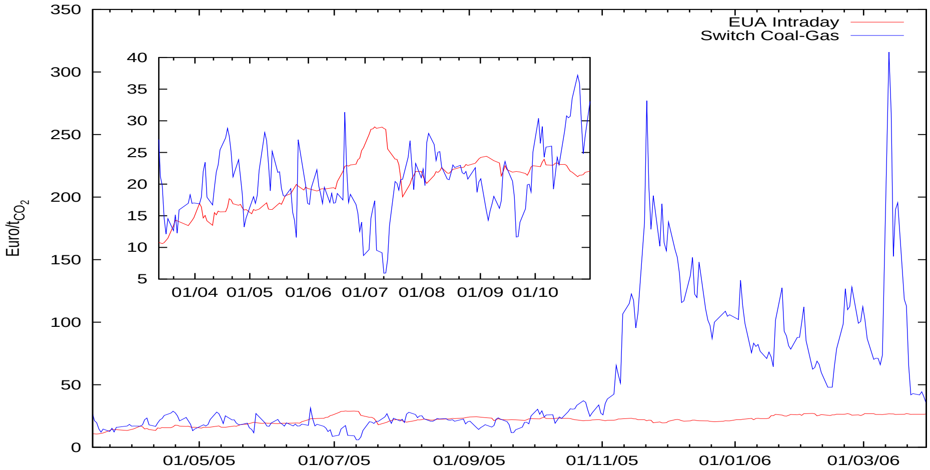

Figure 2: The price for EUA versus fuel switching price calculated for gas/coal spot prices.

The strategy to perform fuel switch helps to meet the emission cap, since instead of the business-as-usual allowance demand Γ^i merely $\Gamma^i - \sum_{t=0}^{T-1} \xi_t^i$ ton of carbon dioxide are to be covered at the end of the compliance period. Thus, we correct Γ^i in the equation (6) by $\Gamma^i - \sum_{t=0}^{T-1} \xi_t^i$ which with (2) expresses the win/loss of the producer i by

$$I_T^{\theta^i,\xi^i,A,i} = V_T^{\theta^i,A} - \theta_T^i A_T - \pi(\Gamma^i - \sum_{t=0}^{T-1} \xi_t^i - \theta_T^i)^+ - \sum_{t=0}^{T-1} \xi_t^i \mathcal{E}_t^i. \tag{7}$$

Assumption To avoid problems in existence of expected values for (7) and (13), we suppose that

$$\Gamma^i, \mathcal{E}_t^i \text{ are integrable for } i = 1, \dots, N, \ t = 0, \dots, T-1. \tag{8}$$

We substantiate further argumentation using Banach spaces L_1 and L_∞ of $P-$integrable and essentially bounded $\mathcal{F}_T-$measurable random variables respectively. Further, we introduce the following spaces of adapted processes

$$\begin{aligned}
\mathcal{L}_1 &:= \{(\Xi_t)_{t=0}^{T-1} : \Xi_t \in L_1, t = 0, \dots, T-1\} \\
\mathcal{L}_\infty &:= \{(\xi_t)_{t=0}^{T-1} : \xi_t \in L_\infty, t = 0, \dots, T-1\} \\
\mathcal{U}_i &:= \{(\xi_t^i)_{t=0}^{T-1} : [0, \lambda_i]\text{-valued process}\} \\
\mathcal{U} &:= \times_{i=1}^N \mathcal{U}_i
\end{aligned}$$

Following the conception that each market participant maximizes the own wealth by trading allowances and applying fuel switching, we formulate the individual optimization problem, given the fuel switch process $A = (A_t)_{t=0}^T$ as

$$\mathcal{L}_1 \times L_1 \times \mathcal{U}_i \to \mathbb{R}, \qquad (\theta^i, \xi^i) \mapsto E(I_T^{A,i}(\theta^i, \xi^i)). \tag{9}$$

With these notations, the equilibrium definition is introduced as

Definition 1. *Given fuel switching price process* $\mathcal{E} = ((\mathcal{E}_t^i)_{i=1}^N)_{t=0}^{T-1} \in \mathcal{L}_1^N$ *of agents* $i = 1, \ldots, N$*, the price process* $A^* = (A_t^*)_{t=0}^T$ *is called equilibrium carbon price process, if there exists* $(\theta^{i*}, \xi^{i*}) \in \mathcal{L}_1 \times L_1 \times \mathcal{U}_i$ *for* $i = 1, \ldots, N$ *with*

$$\begin{array}{l} E(I_T^{A^*,i}(\theta^{i*}, \xi^{i*})) \geq E(I_T^{A^*,i}(\theta^i, \xi^i)) \quad \text{for all} \\ (\theta^i, \xi^i) \in \mathcal{L}_1 \times L_1 \times \mathcal{U}_i, \;\; i = 1, \ldots, N \end{array} \tag{10}$$

such that financial positions are in zero net supply

$$\sum_{i=1}^N \theta_t^{*i} = 0 \quad \text{at any time } t = 0, \ldots, T. \tag{11}$$

Remark It should be emphasized that zero net supply (10) is stated at $t = 0, \ldots, T-1$ for another reason than at $t = T$. For $t = 0, \ldots, T-1$ we have agreed that $(\theta_t^{*i})_{i=1}^N$ are futures positions, whereas at maturity, $(\theta_T^{*i})_{i=1}^N$ stand for the *change* in the initial physical allocation of the agents $i = 1, \ldots, N$.

It turns out that the above equilibrium notion enjoys the property of social optimality. Namely, we show that equilibrium in the above sense automatically results in the solution of a certain global optimization problem, where the total pollution is reduced at minimal overall costs. Beyond the economical interpretations of social-optimality, the importance of the global optimization is that its solutions helps to show the equilibrium existence and to calculate the corresponding carbon prices. Let us explore this connection.

Suppose we are given the fuel switching prices $\mathcal{E} \in \mathcal{L}_1^N$ of agents $i = 1, \ldots, N$. For the switching policy $\xi \in \mathcal{U}$, we denote the final overall switching costs by

$$F(\xi) = \sum_{i=1}^N \sum_{t=0}^{T-1} \xi_t^i \mathcal{E}_t^i.$$

Further, write

$$\Pi(\xi) \;\; = \;\; \sum_{i=1}^N \sum_{t=0}^{T-1} \xi_t^i, \tag{12}$$

for the entire saved carbon, when following policy ξ and denote by

$$\Gamma = \sum_{i=1}^N \Gamma^i$$

the overall allowance demand. Finally, express the total objective from fuel switching and penalty payments as

$$G(\xi) = -F(\xi) - \pi(\Gamma - \Pi(\xi))^+, \quad \xi \in \mathcal{U}, \tag{13}$$

and introduce the global optimization problem

$$E(G(\xi^*)) = \sup_{\xi \in \mathcal{U}} E(G(\xi)), \tag{14}$$

where a switching policy $\xi^* \in \mathcal{U}$ for all agents is to be determined, which minimizes the social costs of non-compliance.

The existence of ξ^* is ensured by standard arguments presented in section 4.1

Proposition 1. *With above notations and assumptions, there exists a solution $\xi^* \in \mathcal{U}$ to the global optimal control problem (14).*

It turns out that the equilibrium carbon price equals to the marginal contribution of an extra allowance to lower the potential payment in the case of non-compliance

$$A_t^* \quad = \quad -\frac{\partial}{\partial x} E(\pi(\Gamma - \Pi(\xi^*) - x)^+ \,|\, \mathcal{F}_t)$$

provided that the global-optimal fuel switching policy $\xi^* \in \mathcal{U}$ from (13) is followed. However, the precise formulation is given in the main theorem below, whose proof is based on additional results, and therefore, is addressed in a separate Section 4.1.

Theorem 1. *Suppose that*

$$\begin{array}{c}\textit{the } \mathcal{F}_{T-1}\textit{-conditional distribution of } \Gamma \\ \textit{posseses almost surely no point mass.}\end{array} \tag{15}$$

With above notations and assumptions, the equilibrium carbon price process is given in terms of global-optimal policy $\xi^* \in \mathcal{U}$ from the Proposition 1 by

$$A_t^* \quad = \quad \pi E(1_{\{\Gamma - \Pi(\xi^*) \geq 0\}} \,|\, \mathcal{F}_t) \qquad \textit{for } t = 0, \ldots, T. \tag{16}$$

3 Quantitative results

For illustration purposes, this section is devoted to quantitative aspects of carbon market equilibrium. We elaborate on a generic model whose parameters are chosen close to the situation of EU ETS. However, to realize calculations, we have to agree on several simplifications, which are justified through the case-by-case arguments given below.

For simplicity, we neglect the discounting effects by supposing that the interest rate equals to zero. Further, we focus on the one-period market implementing so the theoretical considerations from the previous sections. Note however that such a description does not fully reflect the situation of EU ETS in the period 2005–2007, since a certain amount of allowances could be banked into the next period 2008–2012 and, more importantly, the penalty structure is different from what we model. At the end of the first EU ETS period, one EUA from the second period is charged for non-compliance *in addition* to a fee of 40 EURO. Another simplification is that we do not consider the impact of allowances gained from CDM- and JI-projects. This assumption seems maintainable, since the allowance demand reduction resulting from CDM and JI is well-forseen, hence it is captured by the anticipated demand dynamics $(E(\Gamma|\mathcal{F}_t))_{t=0}^T$. However, more difficile is that we consider a single fuel switch price process. The argument therefore is that in order to save carbon, the cheapest technology is applied first. Thus, given a sufficient amount of the corresponding switch capacity, other technologies are never used. We belive that this situation matches the European electricity business, where the new CCGT-technology (giving 52 % efficiency) captures a notable part of production with more than 100 Giga Watt of installed capacity in 2008. In our calculation, we assume the fuel switch capacity amounts to $\lambda := 570.08$ Mega tonn per day, i.e. a the power of 70 GW from CCGT technology is available 16h a day (since within 8h of peak load time both, the CCGT and coal technology run simultaneously). Next, let us elaborate on the time resolution in our calculations. Since the model is set up in terms of

$$(\mathcal{E}_t)_{t=0}^{T-1}, \quad (\Gamma_t = E(\Gamma|\mathcal{F}_t))_{t=0}^T \tag{17}$$

which represent the evolutions of fuel switch price and of expected demand respectively, the reasonable time unit should not fall below the time required to re-schedule a CCGT turbine. Nonetheless, we put this fact back in favor of using continuous-time pendants

$$(\mathcal{E}(t))_{t\in[0,\mathcal{T}]}, \quad (\Gamma(t))_{t\in[0,\mathcal{T}]} \tag{18}$$

for fuel switch price and expected demand evolutions. Note that we write the time parameter in parenthesizes instead using subscript, to indicate continuous-time processes. Moreover, the horizon for continuous time is $[0, \mathcal{T}]$, where we suppose that the time unit equals to one year. By sampling (18) at discrete times, we obtain discrete-time versions for (17) on the daily time step resolution.

Fuel switch process Since during the pre-Kyoto period 2005 – 2007 at least half of the entire EU fuel switch capacity is located within the United Kingdom, we have decided to base fuel switch prices on the McCloskey North-West Europe Steam Coal Index and on natural gas prices from NBP (National Balancing Point, which specifies delivery location within the UK). The continuous–time fuel switch price process is modeled by

$$\mathcal{E}(t) = P(t) + X(t) \qquad t \in [0, \mathcal{T}] \tag{19}$$

where the deterministic part

$$P(t) = a + bt + \sum_{j=0}^{2} c_j \cos(2\pi\varphi_j t + l_j) \qquad t \in [0, \mathcal{T}] \tag{20}$$

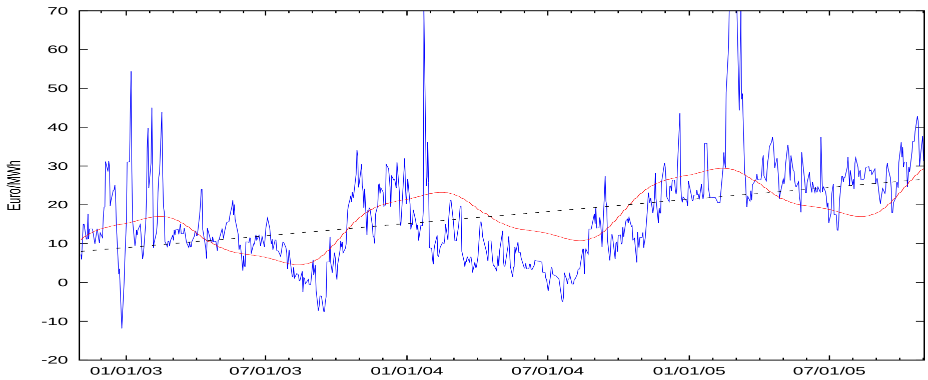

Figure 3: Historical fuel switch prices for CCGT technology calculated with (5) and based on historical data from McCloskey Index and NBP natural gas spot prices.

accounts for a linear price increase superposed by seasonal price fluctuations. The stochastic part $(X(t))_{t\in[0,\mathcal{T}]}$ is modeled by an Ornstein-Uhlenbeck process whose evolution follows the stochastic differential equation

$$dX(t) = \gamma(\alpha - X(t))dt + \sigma dW(t) \tag{21}$$

driven by Brownian motion $(W(t))_{t\in[0,\mathcal{T}]}$ with parameters $\gamma, \alpha, \sigma \in \mathbb{R}$. After performing estimation (for details, we refer the reader to the Section 4.2) based on historical data depicted in the Figure 3, the process (19) is identified with the following parameters:

stochastic part $(X(t))_{t\in[0,\mathcal{T}]}$		
γ	α	σ
31.82	-0.12	68.24

(22)

deterministic part $(P(t))_{t\in[0,\mathcal{T}]}$										
a	b	c_0	φ_0	l_0	c_1	φ_1	l_1	c_2	φ_2	l_2
21.42	6.19	7.62	1	5.95	0.55	2	1.14	1.11	3	3.24

(23)

Expected allowance demand The continuous-time pendant for the expected allowance demand is described by

$$\Gamma(t) := m + vW'(t) \qquad t \in [0, \mathcal{T}]$$

where $(W'(t))_{t\in[0,\mathcal{T}]}$ follows a Brownian motion independent of $(W(t))_{t\in[0,\mathcal{T}]}$ in (21). In this context, the parameters m and v are interpreted as the mean and the standard deviation (times $\mathcal{T}$) of the final allowance demand. In accordance with market data for EU ETS we have set $m = 52$, $v = 52$ which are expressed in Mega tons of carbon dioxide.

Numerical implementation As mentioned earlier, both processes $(\mathcal{E}(t))_{t\in[0,\mathcal{T}]}$ and $(\Gamma(t))_{t\in[0,\mathcal{T}]}$ are interpreted as continuous time pendants of fuel switch prices and expected allowance demand respectively. For numerical calculations, we have applied the standard trinomial tree discretization of each component of the two-dimensional diffusion process $(\mathcal{E}(t),\Gamma(t))_{t\in[0,\mathcal{T}]}$ to solve the corresponding dynamic optimization problem (14) through backward induction method. The Figure 4 graphically illustrates this technique. At each node, we see a splitting into three

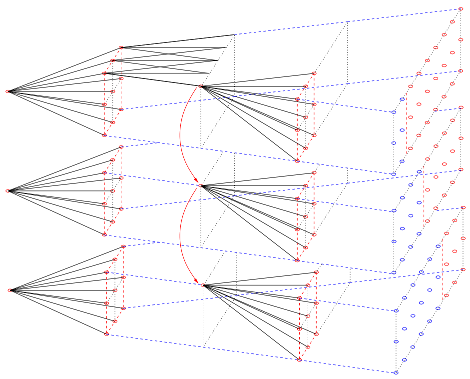

Figure 4: The optimal control by backward induction, applied to the trinomial tree discretization to each component of the processes $(\mathcal{E}(t),\Gamma(t))_{t\in[0,\mathcal{T}]}$.

vertical and three horizontal directions, giving nine branches in all. The vertical direction describes the movement of the fuel switch price, whereas the horizontal illustrates the expected demand dynamics. At maturity time, paths disembogue either at positive realizations of the allowance demand (blue points) or at non-positive (red points). The optimally controlled fuel switch process is calculated by backward induction: At each node the maximum principle is applied to decide either to apply the fuel switch or not. If the fuel switch is performed, then the state is changed due to the effectively reduced allowance demand, indicated by a move to the next lower tree in the forest diagram.

We now discuss the impact of parameters on carbon price. The following numerical illustration is based on discrete-time model $(\mathcal{E}_t)_{t=0}^{T-1},(\Gamma_t)_{t=0}^{T}$ which corresponds to the parameters (22), (23) estimated in Section 4.2. Here, time horizon $0,\dots,T=253$ stands for the year 2005 (the fuel switch process is based on the deterministic component from this year and the starting point $\mathcal{E}_0$ is set at the value of the deterministic component at the beginning of January 2005).

Present values For commensurability reasons, we have decided to show the dependence of allowance price A_t^* on allowance demand Γ_t in terms of the *relative* *demand*

$$\delta_t = \frac{\Gamma_t - \sum_{s=0}^{t} \xi_s}{\lambda(T-t)}, \quad t = 0, \dots, T-1$$

which stands for the percentage of time steps at which the fuel switch at full intensity is needed in order to meet the compliance. The dependence illustrated in the Figure 5 is obvious. The price A_t^* is increasing in δ_t and $\mathcal{E}_t$. Moreover, for $\delta_t \to +\infty$, the allowance price approaches the boundary π of 40 EURO, whereas

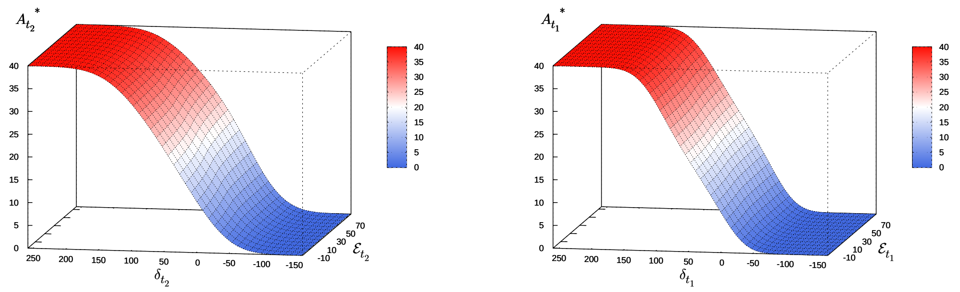

Figure 5: The dependence of allowance price on the present expected demand δ_t and $\mathcal{E}_t$ for different times (right: $t = t_1$ beginning of March, left: $t = t_2$ beginning of September).

for $\delta_t \to -\infty$ it tends to 0. Further, A_t^* changes significantly with moderate deviations in δ_t. On the contrary, the impact of the present fuel switch price $\mathcal{E}_t$ is weak due to the distinct mean reversion. This poor correlation between instantanuous fuelswitch price and allowance price is accurate also in reality as can be observed in Figure 2. Despite the low dependence of A_t^* on $\mathcal{E}_t$, we suppose though that fuel switch price is a significant factor, whose impact is effected through the expected long term fuel switch prices (to be deduced from fuel futures, whose dynamics is not modeled here).

Dependence on model parameters In accordance, the left picture in Figure 6 shows high sensitivity of allowance price on α, which settles the level of expected long term fuel prices. For this reason, we have decided to visualize the effect of α by a plot of A_0^* against $E(\sum_{t=0}^{T-1} \mathcal{E}_t)/T$. Moreover, this figure shows a weak dependence of allowance price on σ, which we illustrate by a plot of A_0^* against the stationary fuel switch price variance $\sigma^2/(2\gamma)$. The right picture in the Figure 6 shows that dependence of A_0^* with respect to changes in $E(\Gamma)/(\lambda T)$ is higher than in v. In other words, the dependence of allowance price on the need for emission reductions is high whereas the uncertainty about necessitative emission reduction is of secondary importance.

Let us summarize our findings. The allowance price should be significantly correlated to the expected long term fuel prices and to the expected need for emission reduction amount. That are the main price drives, since the remaining factors (recent fuel switch prices, their volatility, uncertainty on the required emission reduction amount) have a minor effect on the carbon price formation.

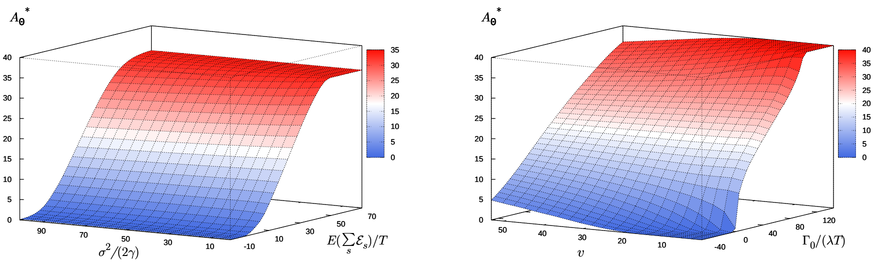

Figure 6: On the left: the impact of α and σ on allowance price, expressed for $\gamma = 31.82$ through long-term fuel switch price mean $E(\sum_{t=0}^{T-1} \mathcal{E}_t)/T$ and variance $\sigma^2/(2\gamma)$ respectively. On the right: the impact of v and $E(\Gamma)/(\lambda T)$.

Regulatory aspects Designing a legally binding scheme, one of the main concerns for regulatory authority is on one hand, to fulfill environmental targets (at least with a certain probability) and, on the other hand, to achieve this emission reduction at the lowest costs for the final consumer. Thus, we have studied the dependence of compliance probability and allowance price on the penalty level and on the initial expected allowance demand (note that this value is controlled by the total amount of allocated allowances). The diagrams in the Figure 7 show the corresponding calculations. Again we show this influence in terms of the relative demand $\delta_0 = E(\Gamma)/(\lambda T)$, which stands for the percentage of time steps at which the fuel switch at full intensity is needed in order to meet the initially expected allowance demand. One concludes that up to the relative demand of 50% the penalty can be increased without a notable increase of the allowance price, giving, however a strong *increase of the compliance* probability. If the relative demand is far above 50%, then the situation changes. The moderate increase of compliance probability is reached only at the expenses of high allowance price. Note that the initial allowance price is directly related to the consumers costs since EUA's are added to electricity prices as an extra consumed commodity.

Derivatives payoff In this approach, we do not discuss risk neutral valuation of carbon derivatives, since the corresponding risk neutral dynamics has not been

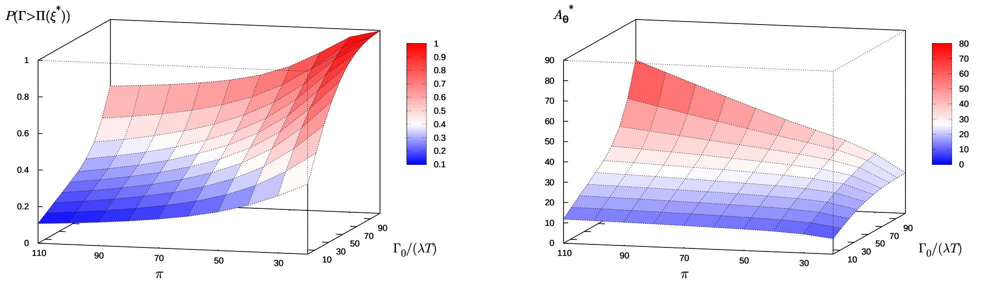

Figure 7: The probability of non-compliance and the initial allowance price depending on penalty size and fuel switch demand.

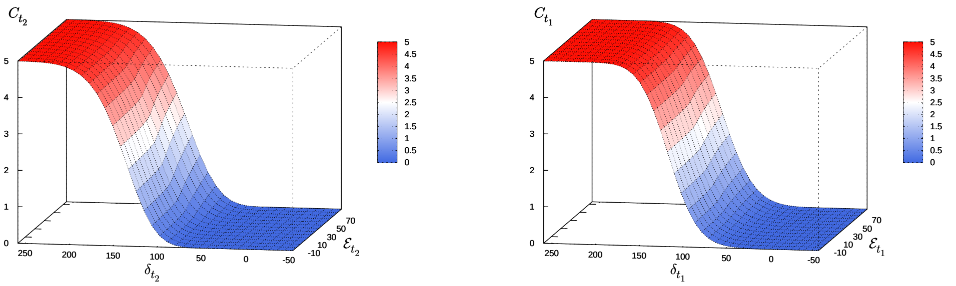

Figure 8: Expected payoff of the European Call with strike price 35 EURO with maturity at the beginning of October at different times; $t = t_1$ beginning of March, $t = t_2$ beginning of September. The call payoff is plotted against the recent fuel switch price $\mathcal{E}_t$ and the relative allowance demand δ_t.

addressed. However, the practice of weather derivatives has shown that an estimation of derivatives payoffs is also important in risk management. Figure 8 shows the expected payoff of a European Call. Again, we observe a weaker dependence on the initial fuel switch price than on the allowance demand.

4 Appendix

4.1 The equilibrium carbon price process

To show that the equilibrium carbon price is connected to the solution of the global control problem (14) via (16), the following re-parameterization is required.

Given the fuel switching policy $(\xi^i_t)_{t=0}^{T-1} \in \mathcal{U}_i$ of the agent i, replace the de facto allowances trading $(\theta^i_t)_{t=0}^T \in \mathcal{L}_1 \times L_1$ by the virtual trading $(\vartheta^i_t)_{t=0}^T \in \mathcal{L}_1 \times L_1$ given by

$$\vartheta^i_t = \theta^i_t + \sum_{s=0}^{t} \xi^i_s, \quad t = 0, \ldots, T. \tag{24}$$

Next, for $(\vartheta^i_t)_{t=0}^T \in \mathcal{L}_1 \times L_1$, $(\xi^i_t)_{t=0}^{T-1} \in \mathcal{U}_i$ introduce the objective

$$\mathcal{I}^{A,i}_T(\vartheta^i, \xi^i) \;\; := \;\; V^{\vartheta^i,A}_T - \vartheta^i_T A_T - \pi(\Gamma^i - \vartheta^i_T)^+ + \textstyle\sum_{t=0}^{T-1} \xi^i_t (A_t - \mathcal{E}^i_t). \tag{25}$$

which, as shown in the proof of the following proposition, expresses (7) in terms of the virtual trading in the sense that

$$\text{if } (\xi^i_t)_{t=0}^{T-1},\ (\vartheta^i_t)_{t=0}^T,\ (\theta^i_t)_{t=0}^T \text{ fulfill (24), then} \quad I^{A,i}_T(\theta^i, \xi^i) = \mathcal{I}^{A,i}_T(\vartheta^i, \xi^i). \tag{26}$$

Consequently, we have the following equilibrium characterization for the new parameterization (compare with Definition 1)

Proposition 2. *$A^* = (A^*_t)_{t=0}^T$ is an equilibrium carbon price process, if for $i = 1, \ldots, N$ there exist $(\vartheta^{i*}, \xi^{i*}) \in \mathcal{L}_1 \times L_1 \times \mathcal{U}_i$ satisfying*

$$\begin{array}{l} E(\mathcal{I}^{A^*,i}_T(\vartheta^{i*}, \xi^{i*})) \geq E(\mathcal{I}^{A^*,i}_T(\vartheta^i, \xi^i)) \quad \textit{for all} \\ (\vartheta^i, \xi^i) \in \mathcal{L}_1 \times L_1 \times \mathcal{U}_i, \;\; i = 1, \ldots, N \end{array} \tag{27}$$

and

$$\sum_{i=1}^{N} \vartheta^{*i}_t = \sum_{i=1}^{N} \sum_{s=0}^{t} \xi^{*i}_s \quad \textit{holds at each time } t = 0, \ldots, T. \tag{28}$$

Proof. Since the re-parameterization mappings

$$\mathcal{L}_1 \times L_1 \times \mathcal{U}_i \to \mathcal{L}_1 \times L_1 \times \mathcal{U}_i, \quad (\theta^i, \xi^i) \to (\vartheta^i, \xi^i)$$

from (24) are bijections for all $i = 1, \ldots, N$ and $(\theta^i, \xi^i)_{i=1}^N$ fulfills (11) if and only if $(\vartheta^i, \xi^i)_{i=1}^N$ satisfies (28), it suffices to prove (26). This assertion is derived as follows:

$$\begin{aligned} V^{\theta^i,A}_T - \theta^i_T A_T &= \sum_{s=0}^{T} \theta_s (A_{s+1} - A_s) - \theta^i_T A_T \\ &= -\theta_0 A_0 + \sum_{s=1}^{T} (\theta_{s-1} - \theta_s) A_s \\ &= -\vartheta_0 A_0 + \sum_{s=1}^{T} (\vartheta_{s-1} - \vartheta_s) A_s + \sum_{s=0}^{T-1} \xi_s A_s \\ &= V^{\vartheta^i,A}_T - \vartheta^i_T A_T + \sum_{s=0}^{T-1} \xi_s A_s, \end{aligned} \tag{29}$$

next, utilize (24) in to obtain

$$\pi(\Gamma^i - \sum_{t=0}^{T-1} \xi_t^i - \theta_T^i)^+ = \pi(\Gamma^i - \vartheta_T^i)^+ \tag{30}$$

and subtract (30) from (29) to verify (26). □

Remark Before entering the proof of the Proposition 1, let us give some feeling about the mechanism responsible for the equilibrium carbon price formation. The objective (25) shows two effects working on opposite directions

i) the impact of carbon price on emission reduction strategies

ii) the overall effect of emission reduction on the carbon price

Namely, the last term in (25) is responsible for i), illustrating that the fuel switching $\xi_t^i > 0$ is optimally performed if and only if the own switching price falls below the market carbon price: $A_t - \mathcal{E}_t^i \geq 0$. This emission reduction, in turn, is connected to ii) through the expected final carbon price A_T. The higher is the current price A_t, the more agents apply fuel switching decreasing so the probability that the market will be short of credits at the end of the compliance period. However decreasing this probability also decreases the expected final need for carbon allowances and consequently lowers the expected final carbon price A_T. Being a financial asset, carbon allowances can not exhibit a significant price decay from A_t to A_T, that is, the fuel switching at time t also diminishes A_t. Thus, there is an equilibrium level to be reached by carbon price at any time. At time t, the equilibrium carbon price depends on the expected overall compliance gap $E(\Gamma \,|\, \mathcal{F}_t)$ and on the expected future prices $(E(\mathcal{E}_s \,|\, \mathcal{F}_t))_{s=t}^{T-1}$ of fuel switching.

We are now prepared to prove the main Theorem 1.

Proof. Due to the Proposition 2, the equilibrium property of $(A_t^*)_{t=0}^T$ can be shown by an explicit construction of $(\vartheta^{i*}, \xi^{i*}) \in \mathcal{L}_1 \times L_1 \times \mathcal{U}_i$, $i = 1, \dots, N$ which fulfill (27) and (28). To proceed so, let $\xi^* \in \mathcal{U}$ be from the Proposition 1 and define by $(\vartheta^{i*})_{i=1}^N \in (\mathcal{L}_1 \times L_1)^N$

$$\begin{aligned} \vartheta_t^{*i} &= \sum_{s=0}^{t} \xi_s^{*i} \quad \text{for all } i = 1, \dots, N,\ t = 0, \dots, T-1, \\ \vartheta_T^{i*} &= \Gamma^i - (\Gamma - \Pi(\xi^*))/N. \end{aligned}$$

The restriction (28) is obviously fulfilled, so we focus on (27).

For $\vartheta \in (\mathcal{L}_1 \times L_1)^N$, $\xi \in \mathcal{U}$, and carbon price processes (16), express (25) as

$$\mathcal{I}_T^{A^*,i}(\vartheta^i, \xi^i) = \sum_{t=0}^{T-1} \vartheta_t^i (A_{t+1}^* - A_t^*) - \vartheta_T^i A_T^* - \pi(\Gamma^i - \vartheta_T^i)^+ + \sum_{t=0}^{T-1} \xi_t^i (A_t^* - \mathcal{E}_t^i) \quad (31)$$

which gives the expected value

$$E(\mathcal{I}_T^{A^*,i}(\vartheta^i, \xi^i)) = E(-\vartheta_T^i A_T^* - \pi(\Gamma^i - \vartheta_T^i)^+) + E(\sum_{t=0}^{T-1} \xi_t^i (A_t^* - \mathcal{E}_t^i))$$

since the process $A^* \in \mathcal{L}_\infty$ follows a martingale by definition (16) and $(\vartheta_t^i)_{t=0}^{T-1}$ is an element from $\mathcal{L}_1$. Thus, to show (27), it suffices to prove that

$$\vartheta_T^i \mapsto E(-\vartheta_T^i A_T^* - \pi(\Gamma^i - \vartheta_T^i)^+) \quad \text{is maximized on } L_1 \text{ at } \vartheta_T^{i*}, \qquad (32)$$

$$\text{whereas} \quad \xi^i \mapsto E(\sum_{t=0}^{T-1} \xi_t^i (A_t^* - \mathcal{E}_t^i)) \qquad \text{is maximized on } \mathcal{U}_i \text{ at } \xi^{i*}. \qquad (33)$$

First, we turn to (32) showing that the maximum is attained pointwise. By (16), $\omega \in \{\Gamma - \Pi(\xi^*) < 0\}$ implies that $A_T^*(\omega) = 0$ and $\vartheta_T^{i*}(\omega) > \Gamma^i(\omega)$. Moreover, the maximum of

$$z \mapsto -z A_T(\omega) - \pi(\Gamma^i(\omega) - z)^+ \qquad (34)$$

is attained on each point from $[\Gamma^i(\omega), \infty[$, thus $\vartheta_T^{i*}(\omega)$ is a maximizer. In the other case, $\omega \in \{\Gamma - \Pi(\xi^*) \geq 0\}$, we have $A_T^*(\omega) = \pi$ and $\vartheta_T^{i*}(\omega) \leq \Gamma^i(\omega)$. Here the maximum of (34) is attained on $[0, \Gamma^i(\omega)]$, thus $\vartheta_T^*(\omega)$ is a maximizer.

Now we turn to (33). It suffices to show that for each $i = 1, \ldots, N$ and $t \in \{0, \ldots, T-1\}$ the following inclusions hold almost surely

$$\{A_t^* - \mathcal{E}_t^i > 0\} \subseteq \{\xi_t^{*i} = \lambda_i\}, \qquad (35)$$

$$\{A_t^* - \mathcal{E}_t^i < 0\} \subseteq \{\xi_t^{*i} = 0\}. \qquad (36)$$

First, we emphasize that the dynamical principle of optimal control implies that

$$\begin{array}{l} \text{for any } \xi \in \mathcal{U} \text{ with } \xi_s = \xi_s^* \text{ for } s = 0, \ldots, t-1. \\ E(G(\xi)|\mathcal{F}_t) \leq E(G(\xi^*)|\mathcal{F}_t) \quad \text{holds almost surely} \end{array} \qquad (37)$$

This assertion is seen by the following argumentation: On the contrary, one uses the $\mathcal{F}_t$–measurable set

$$M := \{E(G(\xi)|\mathcal{F}_t) > E(G(\xi^*)|\mathcal{F}_t)\} \quad \text{of positive measure} \quad P(M) > 0,$$

to outperform ξ^* by ξ' as

$$\xi_s' = 1_M \xi_s + 1_{\Omega \setminus M} \xi_s^* \text{ for all } s = 0, \ldots, T-1. \qquad (38)$$

Note that since ξ and ξ' coincide at times $0,\dots,t-1$, this definition indeed yields an adapted process $\xi' \in \mathcal{U}$. With (38), we have the decomposition

$$G(\xi') = 1_M G(\xi) + 1_{\Omega\setminus M} G(\xi^*),$$

which gives a contradiction to the optimality of ξ^*:

$$\begin{aligned} E(G(\xi')) &= E(E(1_M G(\xi) + 1_{\Omega\setminus M} G(\xi^*)|\mathcal{F}_t)) \\ &= E(1_M E(G(\xi)|\mathcal{F}_t) + 1_{\Omega\setminus M} E(G(\xi^*)|\mathcal{F}_t)) \\ &> E(1_M E(G(\xi^*)|\mathcal{F}_t) + 1_{\Omega\setminus M} E(G(\xi^*)|\mathcal{F}_t)) = E(G(\xi^*)). \end{aligned}$$

To prove (35) and (36) we consider for each λ in the countable set $Q := [0, \lambda^i] \cap \mathbb{Q}$ a strategy $\xi(\lambda, i) \in \mathcal{U}$ defined by

$$\xi_s^k(q,i) = \begin{cases} \lambda & \text{if } s = t \text{ and } k = i \\ \xi_s^{*k} & \text{else} \end{cases},$$

That is, $\xi(\lambda, i)$ coincides with ξ^* with the exception of time t, where only for the agent i the fuel switch intensity is changed from ξ_t^{*i} to a deterministic value $\lambda \in Q$. The policy $\xi(\lambda, i)$ satisfies

$$\begin{aligned} \Pi(\xi(\lambda,i)) &= \Pi(\xi^*) - (\xi_t^{*i} - \lambda) \\ F(\xi(\lambda,i)) &= F(\xi^*) - (\xi_t^{i*} - \lambda)\mathcal{E}_t^i \end{aligned} \quad \text{for all } \lambda \in Q. \tag{39}$$

Since the set Q is countable due to (37), there exists a set $\tilde{\Omega}$ with $P(\tilde{\Omega}) = 1$ such that

$$\frac{E(G(\xi^*|\mathcal{F}_t))(\omega) - E(G(\xi(\lambda,i)|\mathcal{F}_t))(\omega)}{|\xi_t^{*i}(\omega) - \lambda|} \geq 0 \text{ for all } \omega \in \tilde{\Omega} \text{ with } \lambda \neq \xi_t^{*i}(\omega).$$

Using (39) and (13), we conclude from this inequality that

$$\begin{aligned} 0 \leq &-\frac{\xi_t^{*i}(\omega) - \lambda}{|\xi_t^{*i}(\omega) - \lambda|}\mathcal{E}_t^i(\omega) \\ &-E(\pi\frac{(\Gamma_T - \Pi(\xi^*))^+ - (\Gamma_T - \Pi(\xi^*) + (\xi_t^{*i} - \lambda))^+}{|\xi_t^{*i} - \lambda|} \,|\, \mathcal{F}_t)(\omega) \end{aligned} \tag{40}$$

holds for all $\omega \in \tilde{\Omega}$ with $\lambda \neq \xi_t^{*i}(\omega)$. Let us denote the term in in (40) by $D(\xi^*, \lambda)(\omega)$ Approaching $\xi_t^{*i}(\omega)$ by $\lambda \in Q \setminus \{\xi_t^{*i}(\omega)\}$, we apply dominated convergence theorem to obtain

$$\begin{aligned} \lim_{\lambda \uparrow \xi_t^*(\omega)} D(\xi^*, \lambda)(\omega) &= -E\left(\pi 1_{\{\Gamma - \Pi(\xi^*) \geq 0\}} \,|\, \mathcal{F}_t\right)(\omega) \text{ for } \xi_t^{*i}(\omega) \in]0, \lambda^i], \\ \lim_{\lambda \downarrow \xi_t^*(\omega)} D(\xi^*, \lambda)(\omega) &= E\left(\pi 1_{\{\Gamma - \Pi(\xi^*) > 0\}} \,|\, \mathcal{F}_t\right)(\omega) \text{ for } \xi_t^{*i}(\omega) \in [0, \lambda^i[. \end{aligned}$$

Now (15) gives

$$E\left(\pi 1_{\{\Gamma - \Pi(\xi^*) \geq 0\}} \,|\, \mathcal{F}_t\right) = E\left(\pi 1_{\{\Gamma - \Pi(\xi^*) > 0\}} \,|\, \mathcal{F}_t\right) = A_t^*$$

which with (40) implies that the following inclusions hold almost surely: Calculating left limit $\lambda \uparrow \xi_t^i(\omega)$, we have

$$\{\xi_t^{*i} \in]0, \lambda^i]\} \subseteq \{A_t^* - \mathcal{E}_t^i \geq 0\} \;\; \Leftrightarrow \;\; \{A_t^* - \mathcal{E}_t^i < 0\} \subseteq \{\xi_t^{*i} = 0\} \tag{41}$$

For the right limit $\lambda \downarrow \xi_t^i(\omega)$, we obtain

$$\{\xi_t^{*i} \in [0, \lambda^i[\} \subseteq \{A_t^* - \mathcal{E}_t^i \leq 0\} \;\; \Leftrightarrow \;\; \{A_t^* - \mathcal{E}_t^i > 0\} \subseteq \{\xi_t^{*i} = \lambda^i\}. \tag{42}$$

The assertions (41) and (42) give (35) and (36) respectively. □

Finally, it remains to show the existence of the solution to the global optimization problem (14) formulated in the Proposition 1.

Proof. Let us utilize techniques from functional analysis. First, note that $\mathcal{L}_1^N$, equipped with the norm

$$\|\Xi\| = \sum_{t=0}^{T-1} \sum_{i=1}^{N} E(|\Xi_t^i|)$$

is a Banach space with dual $\mathcal{L}_\infty^N$. The canonical bilinear form is

$$\langle \Xi, \xi \rangle := \sum_{t=0}^{T-1} \sum_{i=1}^{N} E(\Xi_t^i \xi_t^i) \quad \Xi \in \mathcal{L}_1^N, \xi \in \mathcal{L}_\infty^N.$$

Next, we consider the weak topology $\sigma(\mathcal{L}_\infty^N, \mathcal{L}_1^N)$ on $\mathcal{L}_\infty^N$ (see [12]). Note that in this topology, the neighborhood basis of a point $\xi \in \mathcal{L}_\infty^N$ is given by all finite intersections of sets

$$B_\xi(\Xi, \delta) := \{\xi' \in \mathcal{L}_\infty^N \, : \, |\langle \Xi, \xi' - \xi \rangle| < \delta\}, \qquad \Xi \in \mathcal{L}_1^N, \, \delta > 0. \tag{43}$$

In other words, $\sigma(\mathcal{L}_\infty^N, \mathcal{L}_1^N)$ is the weakest topology for which any linear functional

$$\mathcal{L}_\infty^N \to \mathbb{R}, \quad \xi \mapsto \langle \Xi, \xi \rangle, \quad \Xi \in \mathcal{L}_1^N \tag{44}$$

is continuous. A function $f : \mathcal{L}_\infty^N \to \mathbb{R}$ is lower semicontinuous at ξ if for each $\varepsilon > 0$ there exist a neighborhood B_ξ of ξ such that $f(\xi') > f(\xi) - \varepsilon$ for all $\xi' \in B_\xi$, the function is called lower semicontinuous, if it is lower semicontinuous at each point. This generalization of continuity is useful since lower semicontinuous functions attain their minima on compact sets. We use this property to show the existence of ξ^*, being a minimizer of $\xi \mapsto E(-G(\xi))$ on $\mathcal{U}$. To proceed so, we need to prove that $\xi \mapsto E(-G(\xi))$ is lower semicontinuous with respect to $\sigma(\mathcal{L}_\infty^N, \mathcal{L}_1^N)$. Given $\xi \in \mathcal{L}_\infty^N$, write

$$E(-G(\xi)) \;\; = \;\; \sum_{t=0}^{T-1} \sum_{i=1}^{N} E(\mathcal{E}_t^i \xi_t^i) + \pi E((\Gamma - \Pi(\xi))^+).$$

Since the first term is a continuous linear functional of the type (44) (evaluated at ξ), it suffices to discuss the lower semicontinuity of the remaining expression

$$\xi \mapsto E((\Gamma - \Pi(\xi))^+).$$

Fix the point ξ and consider for $\xi' \in \mathcal{L}_\infty^N$ the estimate

$$\begin{aligned} (\Gamma - \Pi(\xi'))^+ &\geq (\Gamma - \Pi(\xi'))1_{\{\Gamma-\Pi(\xi)\geq 0\}} \\ &\geq (\Gamma - \Pi(\xi))1_{\{\Gamma-\Pi(\xi)\geq 0\}} - (\Pi(\xi') - \Pi(\xi))1_{\{\Gamma-\Pi(\xi)\geq 0\}} \\ &\geq (\Gamma - \Pi(\xi))^+ - (\Pi(\xi') - \Pi(\xi))1_{\{\Gamma-\Pi(\xi)\geq 0\}}, \end{aligned}$$

thus

$$\begin{aligned} E((\Gamma - \Pi(\xi'))^+) &\geq E((\Gamma - \Pi(\xi))^+) - E((\Pi(\xi') - \Pi(\xi))1_{\{\Gamma-\Pi(\xi)\geq 0\}}) \\ &\geq E((\Gamma - \Pi(\xi))^+) - |\langle \Xi, \xi' - \xi \rangle| \end{aligned} \tag{45}$$

where $\Xi \in \mathcal{L}_1^N$ is given by $\Xi_t^i = E(1_{\{\Gamma-\Pi(\xi)\geq 0\}}|\mathcal{F}_t)$ for all $i = 1, \dots, N$ and $t = 0, \dots, T-1$. Hence, given ε, define the neighborhood $B_\xi(\Xi, \varepsilon)$ of ξ as in (43) which ensures that $|\langle \Xi, \xi' - \xi \rangle| < \varepsilon$ for all $\xi \in B_\xi(\Xi, \varepsilon)$ and finally with (45) yields the lower semicontinuity

$$E((\Gamma - \Pi(\xi'))^+) \geq E((\Gamma - \Pi(\xi))^+) - \varepsilon \quad \text{for all } \xi' \in B_\xi(\Xi, \varepsilon).$$

Let now $(\xi(n))_{n\in\mathbb{N}} \subset \mathcal{U}$ be a sequence approaching the infimum

$$\lim_{n\to\infty} E(-G_T(\xi(n))) = \inf_{\xi\in\mathcal{U}} E(-G_T(\xi)).$$

By the theorem of Banach–Alaoglu, it contains a subsequence $(\xi(n_k))_{k\in\mathbb{N}}$ which converges to ξ^* in the weak topology. Since $\mathcal{U}$ is convex and norm-closed in $\mathcal{L}_\infty^N$, it is also weakly closed, hence $\xi^* \in \mathcal{U}$. Moreover, the semicontinuity ensures that $E(-G_T(\xi^*)) = \inf_{\xi\in\mathcal{U}} E(-G_T(\xi))$. □

4.2 Parameters of fuel switch price process

The present estimation is based on a series of $n = 758$ daily observations

$$(\mathcal{E}(t\Delta)(\omega))_{t=1}^n$$

(where $\Delta = 1/253$ corresponds to one day), which are shown in the Figure 3. The deterministic harmonics (20) in the fuel switch price process are identified with parameters (23) obtained from peaks in the Fourier transform. After removing the deterministic part $(P(t\cdot\Delta)(\omega))_{t=1}^n$ (red line in this figure) the residual component

$$X(t\Delta)(\omega) = \mathcal{E}(t\Delta)(\omega) - P(t\Delta)(\omega), \quad t = 1, \dots, n \tag{46}$$

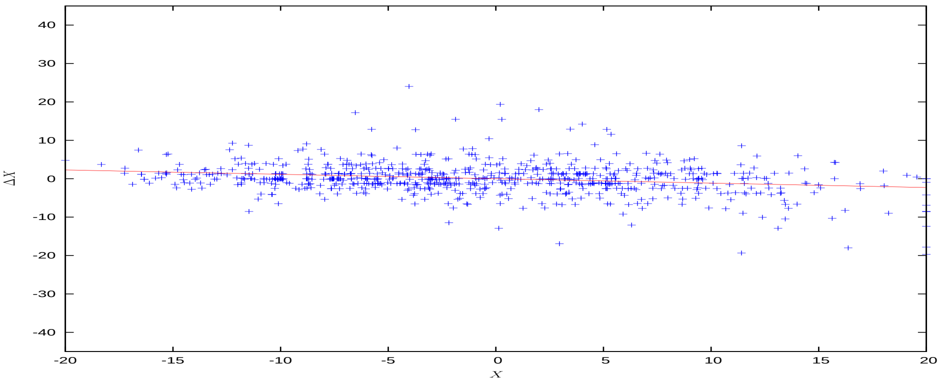

Figure 9: Scatter plot of $(Y(t\Delta)(\omega), X(t\Delta)(\omega))_{t=1}^{n-1}$ calculated by (49) and based on historical fuel switch prices from the Figure 3. The straight line depicts the estimated linear regression.

is modeled as a realization of the Ornstein-Uhlenbeck process (21) whose parameters γ, α, σ are estimated from the data (46) by a standard linear regression method applied as follows: From the formulas for conditional mean and variance

$$\mathrm{E}(X(t)|\mathcal{F}_s) = X(s)e^{-\gamma(t-s)} + \alpha(1 - e^{-\gamma(t-s)}) \quad s \le t \tag{47}$$

$$\mathrm{Var}(X(t)|\mathcal{F}_s) = \frac{\sigma^2}{2\gamma}(1 - e^{-2\gamma(t-s)}) \quad s \le t \tag{48}$$

we obtain the regression

$$Y(t\Delta) := X((t+1)\Delta) - X(t\Delta) = \beta_0 + \beta_1 X(t\Delta) + \beta_2 \epsilon_t \qquad t = 1, \dots, n-1 \tag{49}$$

where $(\epsilon_t)_{t=1}^{n-1}$ are independent, standard Gaussian random variables and $\beta_0, \beta_1, \beta_2$ are connected to α, γ, σ by

$$\begin{aligned} \alpha &= -\frac{\beta_0}{\beta_1} \\ \gamma &= -\frac{1}{\Delta}\ln(1+\beta_1) \\ \sigma &= \sqrt{\frac{2\gamma\beta_2}{1 - e^{-2\gamma\Delta}}} \end{aligned}$$

The Figure 9 shows the plot of $(Y(t\Delta)(\omega), X(t\Delta)(\omega))_{t=1}^{n-1}$, where the maximum likelihood parameter estimate gave $\beta_0 = -0.0147$, $\beta_1 = -0.1182$, $\beta_2 = 16.2708$ from which we have calculated the original parameters α, β, σ displayed in (22).

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