MIT EL 00-005

Energy Laboratory

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The Joint Dynamics of Electricity Spot and Forward Markets: Implications on Formulating Dynamic Hedging Strategies

November 2000

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Energy Laboratory Publication # MIT_EL 00-005

November 2000

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Abstract

The deregulation of the electric utility industry has brought with it a great deal of financial uncertainty for market participants. In this report we address the question of how participants can use available markets in order to mitigate this risk. In order to develop effective strategies for trading, one must first have a good understanding of the dynamics of prices on all available markets. We therefore begin by addressing the relationship between financial and physical, spot and forward markets. In doing so we examine the arbitrage pricing theory approach to modeling forward prices, and evaluate its relevance for non-storable commodities. From the basic relationships between the markets, we arrive at stochastic models which quantify future uncertainty in the marketplace.

Next we consider the case of a load serving entity serving load under a standard offer contract. We show how the stochastic models for load and spot prices allows us to quantify the risk exposure of the LSE. Next we formulate the problem of how an LSE can optimally manage its risk using a periodically rebalanced forward portfolio, based on a mean variance objective function. We show that by using the proposed price models, we can convert the problem into a dynamic programming formulation, which can be solved with a number of computationally efficient tools.

1 Introduction

Competitive power markets exhibit a level of price volatility unparalleled in traditional commodity markets. The reason for this behavior lies in the nature of how electricity is produced and consumed, including lack of storage, inelastic load, and strong seasonal effects on multiple time scales. These characteristics of supply and demand are reflected in the dynamics of market prices, and specifically in the joint dynamics of spot and forward prices. This interaction is of tremendous interest to market participants who wish to use the forward markets to manage their financial risk.

In this report we address the relationship between financial and physical, spot and forward markets. In doing so we examine the arbitrage pricing theory approach to modeling forward prices, and evaluate its relevance for non-storable commodities. From the basic relationships between the markets, we arrive at stochastic models which quantify future uncertainty in the marketplace. These models are then be applied to the problem of dynamic hedging of physical and financial obligations in electricity markets.

2 Power Markets

There are three fundamental markets available for trading electricity. The Spot Market (day ahead), the physical forward or bilateral market, and the financial forward or futures market. While there is no exact mapping between prices across these markets, there is a strong interdependence. We here examine the interplay between the markets, and attempt to define credible models for the joint evolution of prices.

2.1 The Spot Market

The spot market is conducted by either a power exchange or an ISO. Participants submit bids, generally on a day ahead basis, and the market maker clears the market and announces and hourly locational system price. Trade on this market is physical, meaning that physical deliver is always expected. Participants who default on a physical contract will be charged a penalty which is normally dependent on the price of real-time or balancing power in that region.

2.2 The Physical Forward Market

Physical forwards can be traded on an exchange or in a bilateral manner through over the counter (OTC) transactions. Exchange traded forwards use standardized contracts, with power being traded in monthly on- and off-peak blocks (see CBOT definition of power contract). The contract specifies a single MW quantity (q) and a single $/MWh price (F). The short position (seller of the forward contract) is obligated to physically deliver power at a constant rate q to a location specified in the contract (the HUB). The contract does not specify the location at which the power is produced or consumed, but states that the short party is responsible for delivering the power from the generator location to the HUB, and the long position is responsible to deliver the power from the HUB to the load location. For both sides this may involve purchasing additional transmission contracts, or purchasing/selling power through the spot market. Such provisions are not addressed in the contract, and the relative prices of the spot and transmission market will not affect the price of the forward contract.

The price of exchange traded physical forwards is quoted daily by the exchange. The information provided includes the high and low prices as well as the volume traded and the volume of open interest. The exchange quotes prices for every delivery month up to 15 months into the future. This vector of prices G(t), which constantly evolves new trades become public, make up the forward curve for electricity.

$$G(t) = \begin{bmatrix} g_{jan00}(t) \\ g_{feb00}(t) \\ \vdots \\ g_{mar01}(t) \end{bmatrix}$$

Physical forward contracts trade continuously while the exchange is open, until the fourth business day prior to the first delivery day of the contract. At this point trading terminates, and any party left with a short position is required to deliver power according to the provisions in the contract. A trader can avoid this by 'booking out' his position, purchasing a long position which exactly offsets his short position for the same delivery month.

2.3 The Financial Futures Market

Financial futures contracts for electricity are traded on exchanges such as NYMEX and CBOT. Financial contracts are similar to exchange traded physical contract in structure. The main difference is that the parties entering into the contract have no intention of physically producing or consuming the power, but rather use it as a financial hedge against other positions in the market. The financial futures contracts are therefore settled trough the exchange of cash rather than power. In general the payoff function for a party holding the long position in a forward contract is given by:

$$payoff(long) = S_T - F(t,T),$$

where S_T is the spot price at the maturity T, and F(t,T) is the price of the futures contract at the time t it was entered into. The problem which occurs with electricity is that the delivery period for the futures contract is one month, while the underlying spot process is updated on a day ahead basis. As a result, when the futures contract matures on the 4'th business day prior to the first day of the delivery period, the sot prices for hours in the delivery month are not yet known. Hence the contract cannot be settled financially at this time. To circumvent this problem, exchanges have taken on two different approaches, ex-post settling and ex-ante settling.

Ex-post settling: In this approach, the futures contract is settled gradually settled during the delivery month. If two parties have entered into a futures contract for q MWs of on-peak power at a price F, then for every day for the duration of the delivery period, the following process determines the cash flow:

1. The on-peak price of power for the day is calculated by averaging the hourly price of the 16 on-peak hours from the day-ahead spot market. We denote this prices P^{peak}.
2. The long position (buyer of contract) will pay the difference between the P^{peak} and F times the quantity of the contract, times the number of on-peak hours (16). If this quantity is negative then the cash flow will be from the short position to the long position.

The total cash flow for the long position over the duration of the delivery month is given by:

$$\sum_{i=1}^{n} 16q\left(P_i^{peak} - F\right)$$

where n is the total number of days in the delivery period.

Ex-ante settling: In this case, the futures contract is settled financially at its expiration date, ie. on the 4'th day prior to the beginning of the delivery period. Since the day-ahead spot price is not yet known for the delivery month, **the price of a physical forward contract for the same delivery period and location is used in place of the day-ahead spot.** This effectively is a change in the underlying commodity from which the futures contract is derived from a derivative the spot market to a derivative on the physical forward market. The payoff function for the long position at maturity T is given by:

$$\sum_{i=1}^{n} q\left(G(T,T) - F\right)$$

where q is the quantity of the contract in MWs, G(T,T) is the price of a physical forward on the last day of trading, and F is the price at which the futures contract was purchased.

Both the day-ahead spot and physical forward are based on the same commodity, electricity delivered at a specific grid location. However there is no simple mapping between the ex-post average spot price and the ex-ante physical forward price. This is a very crucial point to understand in electricity markets. While the settling procedure differs form market to market, the dominant trend seems to be in the direction of ex-post settling, as seen in California and Nordpool. Unless otherwise specified we will from here on assume that financial forwards settle ex-post.

2.3.1 Arbitrage Pricing and Price Convergence

Arbitrage pricing theory (APT) [6] is based on the belief that pure arbitrage opportunities cannot survive in competitive markets. This assumption imposes constraints on the manner in which prices coevolve in the market. We consider this approach as it relates to three types of assets: stocks, storable commodities, and electricity. We adapt the following definition of arbitrage.

Consider a market with n tradable assets, each with price $X_i(t)$. A portfolio Π is build by purchasing and selling these contracts. The value of the portfolio is given by:

$$\Pi(t) = \sum_{i=1}^{n} w_i(t) x_i(t)$$

where w_i represents the quantity of asset i in the portfolio. W's can be negative if the market allows short-selling. Since future asset values are uncertain, the value of the portfolio at $t>t_0$ is a random variable.

We define an arbitrage opportunity as follows. Arbitrage exists if at time t_0 we can construct a portfolio $\prod$ with the following properties:

$$\Pi(t_0) = 0$$

$$\text{and for some } t > t_0$$

$$\text{Prob}(\Pi(t) < 0) = 0$$

$$\text{Prob}(\Pi(t) > 0) > 0$$

This means that we can construct a portfolio with zero cost, which has zero probability of decreasing in value and a strictly positive probability of increasing in value. Since the portfolio has zero initial cost, any market participant can purchase an unlimited amount of the portfolio, and enjoy a risk free guaranteed profit. The theory is that as arbiters start to take advantage of this opportunity, they will create an upward price pressure on assets with positive weights in the arbitrage portfolio, and downward price pressure on assets with negative weights. Prices will then reach a new equilibrium where the arbitrage opportunity no longer exists.

2.3.2 Application of Arbitrage Pricing Theory (APT) in Electricity Forward and Futures Markets

We now address the relative prices of physical forward and financial futures contracts with ex-post settling, in the framework of arbitrage pricing as defined in the previous section. We allow for the contract to be traded on different exchanges, but assume that there is reasonable price transparency and liquidity in the market. The validity of these addressed at the end of the section.

Recall that the notation for the price of a physical forward contract signed at time t for delivery at time T, is denoted by G(t,T). The equivalent notation for a financial futures contract is F(t,T). We now consider possible relative price levels of the physical and financial markets, and test their consistency with the absence of arbitrage assumption.

First consider the event where at a time t, we observe a set of contracts for delivery month T satisfying the relationship,

$$F(t,T) > G(t,T)$$

A trader can then implement the following strategy.

At time t:

1. Purchase q MW of physical forward contracts.
2. Sell q MW of financial futures contracts.

At time T:

1. For each hour in the delivery period, submit a sell bid of q MW of power into the day ahead spot market at zero price. The power needed to deliver from the spot market is received from the physical forward contract.

The cash flow from this strategy in each time period is shown in the table. Note that all cash flows from forward contracts are realized at the end of the contract.

	t	T
buy physical	0	$NqF(t,T)$
sell financial	0	$NqF(t,T)-\sum_{i=1}^{N} qS_i$
sell spot	0	$\sum_{i=1}^{N} qS_i$
Total	0	$Nq\big(F(t,T)-G(t,T)\big)>0$

This strategy provides a guaranteed profit with zero investment, and therefore it is an arbitrage opportunity which cannot be sustained.

Now consider the case,

$$F(t,T)<G(t,T)$$

The trader adopts the following strategy.

At time t:

1. Purchase q MW of financial futures contracts.
2. Sell q MW of physical forward contracts.

At time T:

1. For every hour in the delivery period, submit a buy bid for q MW to the day-ahead spot market at the market maximum price (we later discuss what happens if the spot market fails to clear). The electricity purchased in the spot market is used to deliver against the obligation from the physical forward contract.

The cash flows in each time period is given by:

	t	T
sell physical	0	- $NqF(t,T)$
buy financial	0	$\sum_{i=1}^{N} qS_i - NqF(t,T)$
sell spot	0	$\sum_{i=1}^{N} qS_i$
Total	0	$Nq\big(G(t,T)-F(t,T)\big)>0$

This strategy provides a guaranteed profit with zero investment, and therefore it is an arbitrage opportunity which cannot be sustained.

The strategies presented above show that in a market free of arbitrage opportunities, the price of a financial forward cannot deviate from the price of a physical forward, in ether a positive or negative direction. This condition must hold true not just at maturity, but during the entire lifetime of the contracts. We thus arrive at the first constraint for electricity derivatives in an arbitrage free marketplace:

$$G(t,T) = F(t,T)\;\forall\; t,T$$

2.3.3 Limits to Arbitrage Pricing Arguments

While APT provides a convincing argument why physical and financial forward prices must be equal at all times, actual observations in the market place show that the two market can diverge at times. The reasons for this inconsistency can be found in the assumptions underlying the arbitrage argument. The following points illustrates how market realities deviate from the theory:

1. **Moving Equilibrium:** Arbitrage pricing theory is based on an equilibrium argument. It states that in a market with active arbitruers, a set of prices which allow for risk-free profit with zero investment is not sustainable. As traders execute the arbitrage, they gradually alter the relative prices until the system settles into an arbitrage free state. Markets in general, and electricity markets in particular, are continuously evolving dynamic systems. The effect is similar to that of a feedback control system driving the states of a system towards a continuously changing control input. Unless the input signal evolves at a significantly slower rate, the states will never settle to their equilibrium value.

In the case of electricity markets, the validity of the equilibrium argument will depend on two factors:

1. The rate at which new information about the future expected value of the spot price enters into the market. Changes in traders perception of the future is the driving input into the futures market. Information which would cause traders to change their perception would include updates on future weather/load conditions, or news of a generator or transmission line outage.

2. The volume and rate at which contracts are trading in the market. This represents the magnitude and speed of the feedback response, or the rate at which the market can react the new information. This is also known as a market's liquidity.

We address this issue in more detail as we introduce our dynamic model for the evolution of the spot price.

2. **Uniqueness of Prices:** Unlike the spot market, the forward markets do not have a unique clearing price. The price quoted by the exchange is a weighted average of all trades in the last day. However there is no guarantee that the trader can find a counter-party willing to trade at exactly this average price at the time the arbitrage is executed. There generally is a gap between the highest price the market is willing to buy, and the

lowest price the market is willing to sell at. This is known as the bid-ask spread. The magnitude of the bid-ask spread is dependant on the liquidity of the market.

3. **Transaction Costs:** Exchanges are generally for profit enterprises. They make a profit by charging a small fee for every contract which is executed on the exchange. The loss incurred by the trader due to such fees is known as a transaction cost. In electricity markets, exchanges generally charge a fixed fee per MWh of power covered in the contract. In order to execute an arbitrage, the guaranteed profit must be greater than the total transaction cost incurred. The magnitude of the transaction cost is relatively minor. Nordpool for example charges approximately one cent per MW traded in a futures contract.

4. **Market Failure:** In designing the arbitrage strategies we assumed that a zero sell bid and max buy bid into the spot market is always accepted. There are situations where the spot market would be unable to deliver additional power at any price due to shortage of generation assets or system security constraints. In such a case the spot market would fail to clear as the aggregate demand and supply curves do now intersect. Under such circumstances there are default conditions specifying the charge/payment to be made to each market participant. In the context of forward markets, the contracts often have a clause for liquidated damages in the case of market failure. The party failing to deliver on a physical obligation must pay whatever financial damage is incurred by the opposing party to replace the power, or any penalty incurred by the opposing party for failing to deliver on its subsequent obligations.

Similar clauses for liquidated damages can be included in financial forwards, thus effectively hedging the trader against market failure.

2.4 Relationship of Spot and Forward Markets

So far we have considered the relationship between physical and financial forward contracts. Under the arbitrage free assumption it could be shown that prices in the two markets have to be equal at all times. Now we consider the relationship between the forward price and the spot price. We apply arbitrage pricing theory to three markets, equity, storable commodities and electricity, to illustrate how the characteristics of the underlying asset changes the pricing model.

2.4.1 The price of a forward contract on a stock

Assume the current price of the stock, which pays no dividends, is S_t and the risk free interest rate is r, continuously compounded. The price of a forward contract on the stock (F(t,T)) with delivery date T must then be $e^{r(T-t)}S_t$. To see why this is true consider the following cases:

1. If $F(t,T) > e^{r(T-t)}S_t$, the investor should sell one forward contract, borrow S_t dollars at the risk free rate (assuming this is possible), and buy one unit of stock. The net cash flow at time t is zero. At time T, the investor delivers the stock against the forward

contract, receives F(t,T) dollars as payment for the forward, and $e^{r(T-t)}S_t$ dollars to pay off his debt. The net cash flow at time T is $F(t,T)-e^{r(T-t)}S_t>0$. This is a pure arbitrage opportunity, which cannot be sustained in an efficient market, and therefore sets the upper limit to the forward price.

2. If $F(t,T)< e^{r(T-t)}S_t$, the investor should buy on forward contract, short-sell one stock, and lend S_t at the risk free rate. The net cash flow at time t is zero. At time T, the investor pays F(t,T) and receives deliver of the stock from the forward contract. He uses this stock to repay his short-selling obligation. He also recovers $e^{r(T-t)}S_t$ from the money lend. The net cash flow is $e^{r(T-t)}S_t-F(t,T)>0$. This is again a pure arbitrage opportunity, setting the lower limit for the forward price.

In this case the upper and lower limits on the forward price are identical, and therefore, in an efficient market where participants can borrow and lend at the risk free rate, the forward price must be given by: $F(t,T)=e^{r(T-t)}S_t$. This illustrates two important points. First, under no-arbitrage conditions, the forward price of a stock is a deterministic function of the spot price and the time to maturity (T-t). Second, there is a smooth convergence of the spot and forward prices at maturity.

2.4.2 The price of a forward contract on a storable commodity

Assume the current unit price of the commodity is S_t, the present value of the total cost of storage incurred during the length of the futures contract is U, and the risk free interest rate is r. The lower bound on the futures price for delivery at time T is $F(t,T)>(S_t+U)e^{r(T-t)}$. If this does not hold, an investor can receive a risk-free profit by borrowing S_t+U at the risk free rate, purchase the commodity and pay off the storage cost, and short a forward contract in the commodity. The cash-flow at time t is zero, and the cash-flow at time T is $F(t,T)-(S_t+U)e^{r(T-t)}>0$. This is known as cash and carry arbitrage.

Payoff at each time step from cash and carry arbitrage:

	t	T
Buy commodity to be delivered against forward contract.	$-S_t$	0
Sell forward contract	0	F(t,T)
Pay storage cost	-U	0
Borrow now, repay at maturity	S_t+U	$-(S_t+U)e^{r(T-t)}$
Total Cash Flow	**0**	$\mathbf{F(t,T) -(S_t+U)e^{r(T-t)}>0}$

Cash and carry arbitrage establishes an upper lower on the forward price of the commodity. The bound converges to the spot price as we reach maturity (T=t), and hence if the forward price is consistently lower than the spot price then the two prices must converge.

The effects of cash and carry arbitrage can also be interpreted as a dynamic relationship between spot and forward prices. Assume that at time t we observe a forward price F(t,T), which violates the upper bound imposed by APT. We would expect the following behavior in the market.

1. In the spot market, demand will increase, as arbitreurs rush to buy the commodity in order to store it. This put upward pressure on the spot market price.
2. On the forward market, the same arbitreurs sell forward contracts in order to execute the arbitrage, creating downward pressure on the forward price.

Now consider the reverse condition, when forward prices drop below spot market levels. In this case, no pure arbitrage strategy is present, since it may not be possible to short sell a physical commodity on the spot market. However, consider the position of a market participant who is currently holding an inventory of the commodity. For this person, the optimal strategy will be to sell the inventory today, and purchase cheap forward contracts which can be used to restore the inventory at a later date. If there is significant inventory in the market, this will put downward pressure on the spot price, and upward pressure on the forward price.

One can question weather the bounds set by APT under realistic market conditions. This is especially true for commodities with thin forward markets and high transaction cost. However, weather or not the bounds are quantitatively accurate, the qualitative interaction between spot and forward prices can certainly be observed.

1. An increase/decrease in the forward price will put upward/downward pressure on the spot price.
2. A spike/drop in the spot price will put upward/downward pressure on the forward price.

Consider the following scenario. Tomorrow OPEC announces that it will reduce its annual production of oil by 50%. Base on these news, the forward price of oil increases sharply. Next, arbitreurs recognize the disparity between spot and forward prices, igniting a buying spree on the spot market. This causes an immediate spike in the spot price of oil. The above scenario illustrates an interesting characteristic of storable commodities markets. **The relationship between the state of physical production and consumption on one hand, and the spot market price on the other, is non-causal.** In other words, a future drop in production leads to a spike in today's spot price. Note however that the relationship between the spot price and the information flow is still causal. That is, the spot market will only react when the drop in future production becomes known to market participants.

The need to model the dynamic relationship between the spot and forward prices in storable commodities has led to the notion of convenience yield (y) which is defined as:

$$F(t,T) = (S_t + U)e^{(r-y)(T-t)}$$

The convenience yields represents the premium the market is willing to pay in order to physically hold the commodity today, rather than a promise for delivery at time T. We can model y as a deterministic parameter, or a stochastic state of the system depending on the market.

2.4.3 The price of a forward contract for electricity

It is easy to see that cash and carry arbitrage is not possible for electricity. To execute the arbitrage one would need to purchase electricity at time t, store it somewhere, and deliver it against a forward contract at time T. Since electricity is not storable one cannot execute this type of arbitrage. As a result, the dynamic relationship between the spot and forward price described above does not hold for electricity. A good example is the case of schedule unit outages. If it were announced today that a major nuclear plant in New England would be out of commission for the month of July, this would cause an immediate increase in today's price of a forward contract with delivery in July. However it would have no effect on the current spot price. We can therefore state that **electricity spot prices are causal in the state of production and consumption of electricity**. This will have a tremendous impact on how we model electricity spot and forward prices.

Without the ability to execute an arbitrage between the spot and forward markets, APT is useless in predicting the relationship between the two markets. Instead we have to address the forces underlying the demand and supply in forward markets. One approach is to assume that the market as a whole is liquid enough that every participant holds a small fraction of the total risk. As a result the market effectively behaves in a risk neutral manner, even if the individual participants are risk averse, allowing us to pose the relationship,

$$F(t,T) = E_t\{S_T\}.$$

The risk neutral formulation is the basis for most risk management and option pricing theories in commodities markets. The problem with this assumption is that electricity markets are relatively illiquid, with a small number of participants. In light of this we here propose a more general model allowing for the existence of a risk premium in the market. We model the forward price as a function of the spot price, the variance of the spot price, and a random disturbance (z^F),

$$F(t,T) = \Phi(E_t(S_T), \mathrm{var}_t(S_T), z^F)$$

The exact structure of the forward risk premium is likely to vary from market to market.

3 Dynamic Hedging

3.1 Motivation

We consider the situation where a load serving entity (LSE) has obligated itself to serve a group of customers at a fixed rate. The contract is set up in such a manner that the customers may consume may consume as much or as little power as they want at any time without additional penalties. This setup is similar to the current 'standard offer' contracts being offered to retail electricity customers. Furthermore we impose the constraint that the LSE owns no generating assets but purchases all power from the spot market. This exposure to the spot market leaves the LSE with significant financial risk. To mitigate this risk it can purchase financial futures contracts on electricity through the commodities exchange. We will here address the problem of how to generate an optimal trading strategy for the LSE in the futures market.

3.2 Problem Formulation

We will now generate mathematical models for the financial risk faced by the load serving entity. This will include modeling the stochastic behavior of loads, spot prices, and futures prices. The notation used is summarized in the table below.

R	Fixed rate for customer (standard offer) ($/MW)
S_k	Spot price in day d.
l_k	Total amount consumed in day d (MW)
$F_{ti,T}$	Price of a forward contract for delivery in month starting at T, as seen at the time of purchase t_i ($/MWh)
$q_{ti,T}$	Total quantity of forward contracts purchased for delivery month starting at T (MW/h) at time of purchase t_i.
M	total number of days in a month
N	number of months in the hedging period
T	Starting day of hedging Period

We begin by modeling the cash flow for the LSE before any purchases in the forward market. This is the unhedged cash flow (CF^U).

$$CF^U = \sum_{m=1}^{N} \sum_{k=Mm+1}^{M(m+1)} l_d (R - S_d)$$

For simplicity we will here consider the case where the hedging period is a single month. The cash flow function then becomes:

$$CF^U = \sum_{d=T}^{T+M} l_d (R - S_d)$$

Next we consider the cash flow incurred from a portfolio of forward contracts $q_{i,m}$. This is the cash flow of the hedge CF^H.

$$CF^H = \sum_{d=T}^{T+M} \left[\sum_{j=0}^{H} q_{t_j,T} (S_d - F_{i,T}) \right]$$

The index t_j represents points in time when we allow the LSE to purchase forward contracts. Since a forward contract $F_{t,T}$ cannot be purchased after the starting date of the delivery month (T), we include the constraint $t \leq T$. The timeline of the hedging process is shown in figure 1.

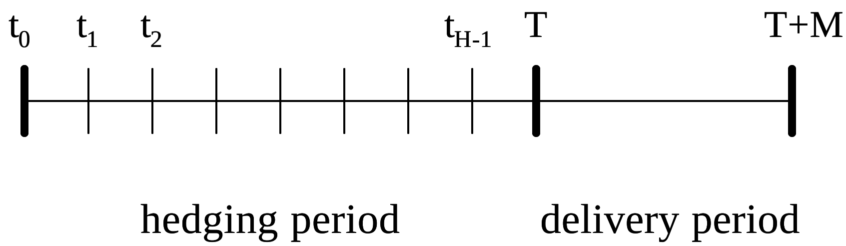

Figure 1

The hedging period [t_0,T] is divided into H hedging intervals of equal length. The number of hedging intervals used will generally depend on the transaction cost and liquidity in the forward market.

Finally we consider the total cash flow for the hedged LSE (CF).

$$CF = \sum_{d=T}^{T+M}\left[l_d (R - S_d) \sum_{j=0}^{H} q_{t_j,T}(S_d - F_{t_j,T}) \right]$$

Not that the forward contracts have no cash flow prior to the delivery period, therefore CF is equal to the total profit received by the LSE.

As seen from the start of the hedging period, l_d, S_d, and $F_{tj,T}$, are all random variables. We therefore define a value function V_{ti} denoting the expected profit seen by the hedger at each step t_i in the hedging period.

$$V_{t_i} = E_{t_i}\{CF\}$$

or in expanded form,

$$V_{t_i} = E_{t_i}\left\{ \sum_{d=T}^{T+M}\left[l_d (R - S_d) + \sum_{j=0}^{i-1} q_{t_j,T}(S_d - F_{t_j,T}) \right] \right\}$$

V_{t0} represents the initial expected return of the unhedged portfolio. The objective of the hedging strategy is to maximize the expected value of the portfolio while minimizing the risk (or variance) of the return. We define a mean variance objective function [8]as,

$$\max_{q_{ti,T}} J = E\{V_T - V_{t_0}\} - rVar\{V_T - V_{t_0}\} .$$

3.3 Underlying Stochastic Models

To solve the stochastic optimization problem formulated above, we require specific information about the joint probability distributions of load, spot prices, and forward prices. To arrive at these distributions we postulate stochastic models for the time evolutions of the random variables. The models described below are simplified versions of the full blown bid based price model described in [1], and generates daily spot prices. The model mimics the supply and demand bids into the spot market. Demand bids are assumed to be inelastic, while the aggregate supply bid curve is modeled as a time varying exponential function. The stochastic processes describing the time evolution of the load and supply states are all Markov, reflecting the temporal relationship between spot market price and supply/demand states discussed in earlier sections. Another key aspect of the model is the link between load and price dynamics. This accounts for the correlation between load and price uncertainty when formulating the hedging strategy.

Spot price Model:

$$S_d = e^{al_d + b_d}$$

Load Model:

$$l_{d+1} - l_d = a^l (m^b - l_d) + s^l z_d^l$$

Supply Model:

$$b_{d+1} - b_d = a^b (m^b - b_d) + s^b z_d^b$$

Forward Price Model:

$$F_{d,T} = E_d \{\overline{S_T}\} + p \operatorname{var}_d \{\overline{S_T}\} + s^F z_d^F$$

where,

$$\overline{S_T} = \sum_{d=T}^{T+N} S_d \ .$$

Note that $\operatorname{var}_t(S_T)$ is a deterministic function of (T-t), and can therefore be thought of as a parameter of the model. Specifically we write:

$$h = [\operatorname{var}_t(S_{t+1}), \operatorname{var}_t(S_{t+2}), ..., \operatorname{var}_t(S_{t+N})]$$

We can express this function in terms of the state vector x_d,

$$x_d^T = [l_d \quad b_d],$$

the output vector y_d,

$$y_d^T = [l_d \quad S_d \quad F_{d,T} \quad V_d],$$

the control variable corresponding to forward market purchases,

$$u_d = q_{d,T}$$

and stochastic inputs,

$$z_d^T = [z_d^l \quad z_d^b \quad z_d^F].$$

We also define a vector of parameters θ,

$$q^T = [m^{l,b} a^{l,b,}, s^{l,b,F}, p, h]$$

We can now write the dynamic constraints,

$$x_{d+1} = Ax_d + Bu_d + Cz_d$$

$$y_d = f(x_d, u_{d,} z_d)$$

Note that while the state dynamics is linear, the output variables are a nonlinear function of the states.

3.4 Properties of the Optimization Problem

Based on the stochastic models described in the previous section, we can describe the properties of the model and the value function V_d. The model is Markov, meaning that all information of future outputs is contained in the current values of the state. As a result the value function V_d is also Markov. The changes in value of V over time is due to changes in the underlying state vector x_d. Furthermore x changes as a function of the noise vector z_d. Since z_d is a stochastic process with independent increments, V_d will also be an independent increment process. We formally write this as:

$$(V_{d+1} - V_d) \text{ is independent of } (V_{d+t+1} + V_{d+t}), \text{ for } t \neq 0$$

Furthermore V_d is a martingale process,

$$E_d(V_{d+t}) = V_d.$$

The proof for this is a simple application of iterated expectations.
Using these characteristics of the value function, we can now rewrite the objective function of the hedging problem,

$$J = E\{V_T - V_0\} - rVar\{V_T - V_0\}.$$

First we write the total change in the value function over the hedging period as a sum of incremental changes,

$$V_T - V_0 = \sum_{i=1}^{N} V_{i+1} - V_i.$$

Since V_d is an independent increment process, the variance becomes a linear function of the increments, and we can write the objective function as,

$$J = \sum_{i=1}^{N} E\{V_{i+1} - V_i\} - r\sum_{i=1}^{N} Var\{V_{i+1} - V_i\}.$$

Furthermore, since V_d is martingale, the increments are zero mean, and we can write,

$$J = \sum_{i=0}^{N} E\{V_i - V_{i-1}\} - r\sum_{i}^{N-1} E\left\{(V_{i+1} - V_i)^2\right\}.$$

Next we join the summation and arrive at the new objective function,

$$J = \sum_{i=1}^{N} E\{V_i - V_{i-1}\} - rE\left\{(V_i - V_{i-1})^2\right\}.$$

Which can be written as:

$$g_d(x_d, w_d, u_d) = (V_d - V_{d-1}) - r(V_d - V_{d-1})^2$$

where g_i is a function of the current state and the disturbance vector. The objective function now becomes,

$$J = E\left\{\sum_{i=1}^{N} g_d(x_d, w_d, u_d)\right\}$$

This problem can be solved recursively by starting at the end of the hedging period,

$$J = E\{g_N(x_N, w_N, u_N)\} + \sum_{i=1}^{N-1} g_d(x_d, w_d, u_d),$$

which conforms to the standard dynamic programming formulation [5]. For this type of formulation there is a wide range of literature regarding efficient solution techniques.

4 Conclusion and Future Work

In this report we show how the unique properties of electricity production and consumption influence the dynamics between electricity spot and forward markets.

Specifically, the price level on the spot market depends only on the current state of demand and supply, and is independent on forward market prices. We apply this understanding of market dynamics to the problem of dynamic hedging of the obligation to serve load under standard offer contracts. A price process is presented which mimics the behavior of load and supply bids into the spot market. It emphasizes the temporal relationship between load and supply states and spot prices. Applying this type of model we show how the dynamic hedging approach can transformed into a standard dynamic programming optimization problem. This result will allow us to apply efficient DP solution techniques to the hedging problem. Future work will focus on solving the dynamic programming problem as posed, and also extend it to the full blown bid based price model as described in [1]. We also want to extend the hedging approach to include risk management by power producers. Specifically, by applying principal component analysis, we can transform the cash flow from a generator with unit commitment constraints, into a Markov type process which lends itself to the techniques outlined in this report [2].

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