

**MIT EL 00-001 WP**

**Energy Laboratory**

**Massachusetts Institute  
of Technology**

# **Implementation of Cluster-based Congestion Management Systems**

**May 2000**

# **Implementation of Cluster-based Congestion Management Systems**

**Yong T. Yoon, Jose R. Arce, Ken K. Collison and  
Marija D. Ilic**

**Energy Laboratory Publication # MIT EL 00-001 WP**

**Energy Laboratory  
Massachusetts Institute of Technology  
Cambridge, Massachusetts 02139-4307**

**May 2000**

# Implementation of Cluster-based Congestion Management Systems

Yong T. Yoon<sup>†</sup>   Jose R. Arce<sup>††</sup>   Ken K. Collison<sup>†</sup>   Marija D. Ilić<sup>†</sup>

<sup>†</sup> Energy Laboratory,  
Massachusetts Institute of Technology,  
Cambridge, MA 02139

<sup>††</sup> Instituto de Energia Electrica,  
Universidad Nacional de San Juan,  
(5400) San Juan, Argentina

To Be Presented At *ICPSOP 2000, "Restructuring The Power Industry For The Year  
2000 And Beyond, 2000*

## **Abstract**

The paper describes the technical challenges in implementing cluster-based congestion management systems. The study attempts to generalize the market-based zonal pricing method being used in the energy market in California for alleviating congestion in the system. The practical implementation of the cluster-based systems consists of two steps: (1) aggregation of individual nodes into clusters and (2) computation of cluster-wide prices. The resulting clusters and prices determine an operating condition that is at the optimum with respect to some pre-defined objective function while keeping the power transfer across cluster interfaces within the acceptable limits. We illustrate two possible ways of aggregating nodes into clusters and compare the resulting solutions.

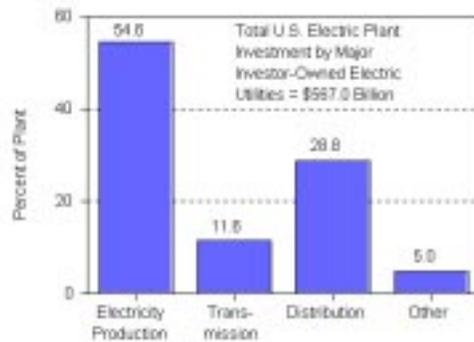
## **Keywords**

Transmission congestion, Nodal pricing, Zonal pricing, Congestion cluster pricing, Congestion Distribution Factors (CDFs)

## I. INTRODUCTION

A simplified modeling of electric power systems comprises of three basic elements: generators that produce the power; the loads<sup>1</sup> that distribute the power for consumption; and the transmission system that connects generators to load centers.

As the electric power industry goes through the deregulation process, the transmission system becomes more important not only from the standpoint of reliability and quality of service, but also from the financial viewpoint.[8] With a little more than \$50 billion of net assets, and around \$3 billion annually in capital expenditures, the transmission constitutes only about 12% of total investment made by utilities as shown in Figure 1.[3] However, by



Source: Energy Information Administration, *Financial Statistics of Major U.S. Investor-Owned Electric Utilities 1996*, DOE/EIA-0437(96/1) (Washington, DC, December 1997).

Fig. 1. Distribution of Utility Investment in 1996

providing interconnecting facilities covering wide geographical areas, transmission plays a key role in fostering competition in wholesale energy markets.

As a physical equipment, each transmission line in a system has operating limits. These limits are usually expressed in terms of power ratings; maximum allowable power that can be transferred through particular lines. These limits on individual lines restrict dispatch of additional generation at specific locations under certain operating conditions. The transmission congestion refers to this inability to dispatch additional generation from certain generators within the system due to transmission line limits.

<sup>1</sup>From the bulk transmission system perspective, the loads consist of distribution companies, electric cooperatives, market aggregators and in some instances, large industrial users.

The presence of transmission congestion in electric grids can significantly limit competition by creating pockets of the system that can only be served by a limited number of generating sources. Plus, when the system operator sets the electricity price and decides the dispatch amount of generators in order to alleviate congestion in the system, the operation of transmission directly affects the individual *profit* by market participants. Thus, in recent years the methods of managing transmission congestion have been under intense scrutiny.[4] [5] [7]

Presently there are two distinct congestion management systems widely being employed: nodal pricing and zonal pricing methods.[6] [9] [11] This paper provides an in-depth examination of implementation of cluster-based congestion management systems. The zonal pricing method is a particular form of cluster-based congestion management system, in which zones refer to the clusters of nodes aggregated based on economic signal. In subsequent sections we present the mathematical derivations necessary for implementing cluster-based congestion management systems and discuss the perspective on the facilitation role of transmission provider in wholesale competition.

The paper is organized as follows:

Section II provides the mathematical background needed for discussion throughout the paper. We then briefly describe the optimization problem to be solved for implementing the nodal pricing method. A detailed description of cluster-based congestion management systems follows in Section III. In particular, two methods, the zonal pricing and congestion cluster methods, are discussed thoroughly. Section IV compares various congestion management systems through illustration and Section V summarizes the conclusions of the paper.

## II. NODAL PRICING METHOD

For the rest of the paper we assume the so-called mandatory system operator model [10] for the electricity market in which the market is conducted by one central dispatcher called the system operator as shown in Figure 2. In this model the system operator sets the electricity price and determines the dispatch amount of each generator in the system based on marginal supply bids submitted by suppliers. The marginal supply bid is an monotonically increasing curve that reflects the supplier's individual preference on the production amount at various prices. Through the price and dispatch amount, the system operator determines the system operating point that is economically optimal while respecting system constraints.

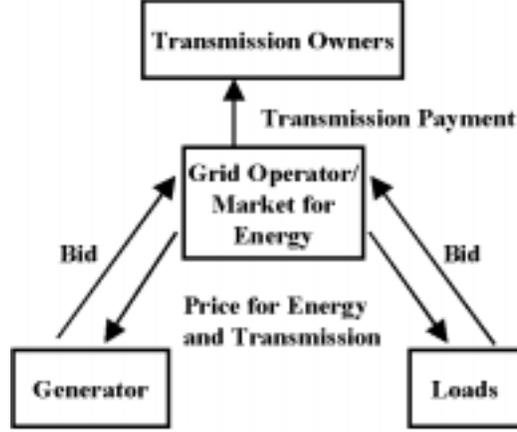


Fig. 2. Mandatory System Operator Model

The nodal pricing method of managing transmission congestion is based on the computation of location marginal price at each individual node in the system developed in [13]. The optimization problem to be solved in order to determine the location marginal prices is given by

$$\min_{Q_{G_i}} \sum_{G_i} C_{G_i}(Q_{G_i}) \quad (1)$$

where

$Q_{G_i}$  : the dispatched generation amount at node  $G_i$

$C_{G_i}$  : the total cost of generation at node  $G_i$  expressed in terms of  $Q_{G_i}$

subject to the load flow constraint, i.e., total generation is equal to system load,

$$\sum_{G_i} Q_{G_i} = \sum_{D_i} Q_{D_i} \quad : \lambda \quad (2)$$

the transmission line flow limit constraints, i.e., the power flow on line  $l$  is within the maximum rating of the line,

$$|F_l| = |H_{lG_i}Q_{G_i} + H_{lD_i}Q_{D_i}| \leq F_l^{max} \quad : \mu_l \quad (3)$$

and the generation limit constraints, i.e., the dispatch amount at node  $G_i$  is within the maximum rating of the corresponding generator

$$0 \leq Q_{G_i} \leq Q_{G_i}^{max} \quad : \eta_{G_i} \quad (4)$$

For simplicity, we use DC power flow in computing the flows on each line in the system. The DC power flow equations in matrix notation are written as:

$$\mathbf{B}\delta = \mathbf{Q}_{G_i} - \mathbf{Q}_{D_i} \quad (5)$$

where

$$\begin{aligned} \delta & : \text{the voltage angle vector} \\ Q_{G_i} & : \text{the real power generation vector for buses } G_i \\ Q_{D_i} & : \text{the real power load vector for buses } D_i \end{aligned}$$

Then the flow vectors for lines can be computed as

$$\mathbf{F}_l = \mathbf{H}\delta \quad (6)$$

where  $\mathbf{H}$  is the linearized flow matrix for the system.

As done in [13], we can solve the optimization problem in (1) by constructing Lagrangian function of the form [2]

$$\begin{aligned} \mathcal{L} = & \sum_{G_i} C_{G_i}(Q_{G_i}) + \lambda \left( \sum_{D_i} Q_{D_i} - \sum_{G_i} Q_{G_i} \right) \\ & + \sum_l \mu_l \left( \sum_{G_i} H_{lG_i} Q_{G_i} + \sum_{D_i} H_{lD_i} Q_{D_i} - F_l^{max} \right) \\ & + \sum_i \eta_{G_i} \left( Q_{G_i} - Q_{G_i}^{max} \right) \end{aligned} \quad (7)$$

where  $\mu_l \neq 0$  if and only if  $F_l = F_l^{max}$ . Taking first derivative of  $\mathcal{L}$  with respect to  $P_{G_i}$  and setting it equal to zero yields

$$\frac{dC_{G_i}}{dQ_{G_i}} + \eta_{G_i} = \lambda + \sum_l \mu_l H_{lG_i} \quad (8)$$

Suppose the generation cost of supplier  $G_i$ ,  $C_{G_i}$ , is a quadratic function of the output given by,

$$C_{G_i}(Q_{G_i}) = a_{G_i} Q_{G_i}^2 \quad (9)$$

Then, under the perfectly competitive market condition, the optimal supply bid by supplier  $G_i$ ,  $B_{G_i}$ , is the marginal cost bid given by,

$$\begin{aligned} B_{G_i} & = \frac{dC_{G_i}}{dQ_{G_i}} \\ & = 2a_{G_i} Q_{G_i} \end{aligned} \quad (10)$$

Matching the solution in Eq. (8) and the supply bid in Eq. (10), the system operator can set the price at node  $G_i$ ,  $\rho_i$ , and the dispatch amount,  $Q_{G_i}$  as

$$\rho_i = \lambda + \sum_l \mu_l H_{lG_i} \quad (11)$$

and

$$Q_{G_i} = \begin{cases} Q_{G_i}^{max} & \rho_{G_i} \geq p_{G_i}^{max} \\ \frac{\rho_i}{2a_{G_i}} & 0 \leq \rho_{G_i} \leq p_{G_i}^{max} \\ 0 & \text{otherwise} \end{cases} \quad (12)$$

where  $p_{G_i}^{max} = 2a_{G_i}Q_{G_i}^{max}$ . Graphically, the dispatched generation amount at node  $G_i$  is uniquely defined under the above assumption once  $\rho_i$  is determined from Eq. (8) as shown in Figure 3. For the generator  $G_i$  shown in the figure, the dispatched generation amount is

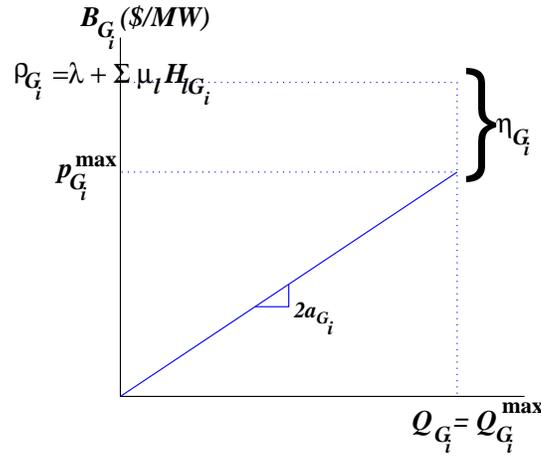


Fig. 3. (Marginal) Supply Bid of Generator  $G_i$

equal to  $Q_{G_i}^{max}$  since  $\rho_{G_i} \geq p_{G_i}^{max}$ .

If there is no transmission congestion present in the system, then Eq. (8) yields

$$\frac{dC_{G_i}}{dQ_{G_i}} + \eta_{G_i} = \lambda \quad (13)$$

which implies that there is no differential in the locational marginal prices. In this case, the marginal price at node  $G_i$  can be determined simply by finding the equilibrium point between the aggregated system supply bid and the system load. Graphically this process can be shown in Figure 4. In Figure 4  $\rho_{G,\text{sys}}$  is the (locational) marginal price of all nodes and  $Q_{G,\text{sys}}$  is the total generation in the system at the equilibrium for the given system

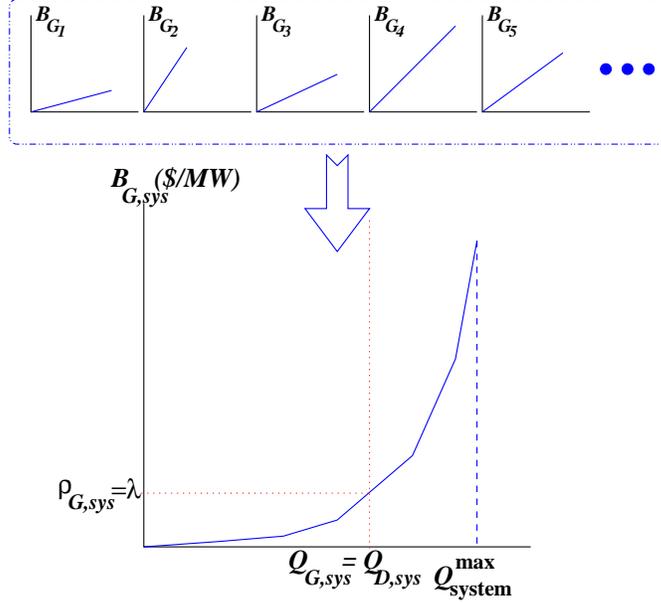


Fig. 4. System Supply Bid under no Transmission Congestion

load  $Q_{G,\text{sys}}$ . The dispatch amount for node  $G_i$  can be computed from  $Q_{G,\text{sys}}$  with respect to  $\rho_{G,\text{sys}}$ . This interpretation of aggregate supply bid is not possible if there is congestion present in the system since each node  $G_i$  takes on different locational marginal prices.

Under the perfectly competitive market condition, the value of congested transmission is reflected in marginal cost of transmission. Given that the locational marginal price in Eq. (11) represents the marginal valuation of net benefits at that node thereby providing the correct generation incentive, and is thus optimal with respect to allocating the limited transmission capacities to the most cost-effective suppliers.

### III. CLUSTER-BASED PRICING METHOD

The strength of deregulation lies upon the utilization of market efficiency through decentralized decision making, customer choices and competition. Thus, in the restructuring of electricity industry it is essential to find the ways to accommodate the execution of physical bilateral contracts through which market participants can rely on decentralized decision making, provide product differentiation with customer choices and foster competition without revealing the proprietary information (such as marginal cost of production) critical for conducting business.

Although the nodal pricing method for managing congestion described in the previous

section results in the most efficient operating point possible at any given instance, under the method the decentralized decision making by individual market participants for implementing physical bilateral contracts is deemed very difficult due to the complexity in internalizing transmission congestion related risks. Even with financial instruments such as Transmission Congestion Contract (TCC) [6], the computation can become quite cumbersome in estimating the transmission congestion risks associated with any bilateral deals, since the risks have to be examined on the individual node basis.

The zonal pricing method being used in the energy market of California was established in response to the above concerns related to physical bilateral contracts. In California the system is divided into a number of smaller markets by aggregating individual nodes into zones whenever there is little expectation of congestion within each market. That is to say, the zones are defined such that the cost related to managing *intrazonal* congestion can be distributed on an average basis since the congestion within any zone is believed to be rare.[1] The marginal cost based *interzonal* pricing and dispatching promotes efficient use as congestion between zones is frequent with large impacts.

Once zones are defined, participants entering into physical bilateral contracts, then, only need to assess the transmission congestion related risks between zones. This reduction in complexity lowers the impediments in entering into bilateral contracts thus fostering competition and customer choices. A greater market efficiency can be achieved through well-functioning secondary markets dealing with financial instruments for congestion related risks since these instruments can be issued with higher accuracy also due to the significant simplification in computation.

The practical implementation of the cluster-based congestion management systems<sup>2</sup> consists of two steps: (1) aggregation of individual nodes into clusters and (2) computation of cluster-wide prices. To the best of author's knowledge, there is little discussion available in the literature on the interplay of the above two steps at the time of this writing. In the subsequent sections zonal pricing method and congestion-cluster pricing method are compared in details in order to study the suboptimality associated with aggregation step of cluster-based

<sup>2</sup>We refer the generalized zonal pricing method to as cluster-based congestion management systems, where the aggregation of nodes into clusters can be done in many different ways other than what is suggested for the electricity markets in California.

congestion management systems relative to the nodal pricing method.

#### A. Zonal Pricing Method

In this scheme the determination of zones is based on price differentials in locational marginal prices. In implementing the method, the system operator first receives the marginal supply bids from suppliers. Using these bids, the optimization problem given in Eq. (1) is solved to locate potentially congested transmission lines. From this result, the system operator computes the locational marginal prices and the corresponding dispatched generation amount at each node  $G_i$ . The nodes with similar nodal prices and geographical proximity are then grouped to form congestion zones.

In this aggregation step the system operator uses his judgment on what level of similarities in nodal prices and what degree of geographical proximities are appropriate for nodes to be grouped together when determining zonal boundaries. The resulting zones must satisfy the number requirement, i.e., no more than  $x$  number of congestion zones in the whole system in order to limit the computational complexity, and the congestion interface requirement, i.e. all potential congested transmission lines identified from solving the initial optimization problem must lie along zonal boundaries.<sup>3</sup>

Suppose the nodes  $G_i, G_{i+1}, \dots, G_{i+k}$  are in the zone  $z_j$ . Then the new generation cost associated with the zone  $z_j$  is given by

$$C_{z_j}(Q_{z_j}) = f_{z_j}(Q_{G_i}, Q_{G_{i+1}}, \dots, Q_{G_{i+k}}) \quad (14)$$

where  $f_{z_j}$  is some monotonically increasing nonlinear function representing the least cost combination of  $Q_{G_i}$ 's in  $z_j$  for producing  $Q_{z_j}$ . Given  $C_{z_j}(Q_{z_j})$  the system operator can now compute the *zonal* price and the corresponding dispatched generation amount at each zone by solving the optimization problem given as

$$\min_{Q_{z_j}} \sum_{z_j} C_{z_j}(Q_{z_j}) \quad (15)$$

subject to the load flow constraint, i.e., total generation is equal to system load,

$$\sum_{z_j} Q_{z_j} = \sum_{D_i} Q_{D_i} \quad : \lambda \quad (16)$$

<sup>3</sup>These zonal boundaries are often referred to as congestion interfaces.

the congestion interface flow limit constraints, i.e., the power flow on any line  $l$  *along only the congestion interfaces* is within the maximum rating of the line,

$$|F_l| = \left| \sum_{z_i} H_{lz_i} Q_{z_i} + \sum_{D_i} H_{lD_i} Q_{D_i} \right| \leq F_l^{max} \quad : \mu_l \quad (17)$$

and the generation limit constraints, i.e., the dispatch amount at zone  $z_j$  is within the sum of maximum rating of the corresponding generators within the zone

$$0 \leq Q_{z_j} \leq \sum_{G_i \in z_j} Q_{G_i}^{max} \quad : \eta_{z_j} \quad (18)$$

Again, we use DC power flow in computing the power flows on line  $l$  along interfaces.

The computation of  $H_{lz_i}$ , however, is not trivial but depends on the composition of  $Q_{z_j}$  in terms of  $Q_{G_i}$ 's where  $G_i \in z_j$ . This composition is related to the generation cost functions of individual generators. Assuming the quadratic cost functions for generation given in Eq. (9) and the corresponding marginal supply bids in Eq. (10), we derive the aggregate marginal supply bid for zone,  $z_j$  as follows:

$$B_{z_j} = \begin{cases} \left( \frac{1}{2a_l} + \frac{1}{2a_{l+1}} + \dots + \frac{1}{2a_{l+s}} \right)^{-1} Q_{z_j} & Q_{z_j} \in R_{I_1} \\ \left( \frac{1}{2a_m} + \frac{1}{2a_{m+1}} + \dots + \frac{1}{2a_{m+t}} \right)^{-1} Q_{z_j} & Q_{z_j} \in R_{I_2} \\ \cdot \\ \cdot \\ \cdot \\ \left( \frac{1}{2a_n} + \frac{1}{2a_{n+1}} + \dots + \frac{1}{2a_{n+u}} \right)^{-1} Q_{z_j} & Q_{z_j} \in R_{I_k} \end{cases} \quad (19)$$

where  $R_{I_i}$ 's define the region of operating condition in zone  $j$  with  $q$  number of generators are still below the generation limits.  $a_r$ 's represent the coefficient of those generators below their generation limits. Since the sensitivity of line flows on zonal injection is given by

$$\frac{dF_l}{dQ_{z_j}} = \frac{dF_l}{dQ_{G_i}} \frac{\partial Q_{G_i}}{\partial Q_{z_j}} + \frac{dF_l}{dQ_{G_{i+1}}} \frac{\partial Q_{G_{i+1}}}{\partial Q_{z_j}} + \dots + \frac{dF_l}{dQ_{G_{i+k}}} \frac{\partial Q_{G_{i+k}}}{\partial Q_{z_j}} \quad (20)$$

with

$$\frac{dF_l}{dQ_{G_i}} = H_{lG_i} \quad (21)$$

the problem reduces to finding the sensitivity of nodal injections to zonal injection to whose zone the node belongs. This sensitivity can be solved by exploiting the relationship between

$Q_{z_j}$  and  $Q_{G_i}$  in the region of operating condition,  $R_{I_i}$  through marginal price as

$$Q_{G_i} = \frac{1}{2a_i} \left( \frac{1}{2a_i} + \frac{1}{2a_{i+1}} + \cdots + \frac{1}{2a_{i+k}} \right)^{-1} Q_{z_j} \quad (22)$$

if  $Q_{G_i} \in R_{I_i}$ .

Graphically, the above derivation has the following interpretation. Without loss of generality we consider a zone consisting only two generators. Given the supply bids at nodes  $G_i$  and  $G_j$ , the aggregate supply bid for zone  $z_k$  can be constructed as shown in Figure 5. For

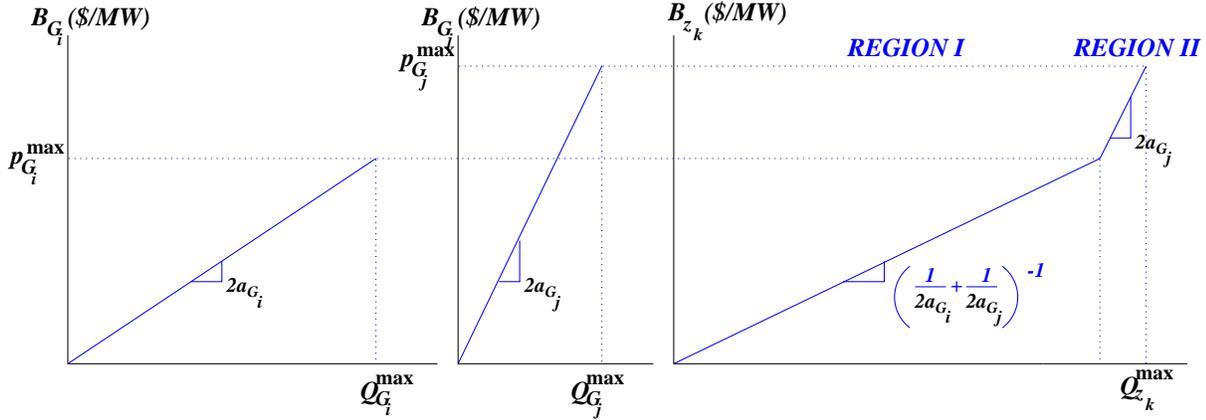


Fig. 5. Aggregation of Marginal Supply Bids in Zone  $k$

Region I

$$\begin{aligned} \frac{dF_l}{dQ_{z_k}} = & \frac{1}{2a_{G_i}} \left( \frac{1}{2a_{G_i}} + \frac{1}{2a_{G_j}} \right)^{-1} H_{lG_i} \\ & + \frac{1}{2a_{G_j}} \left( \frac{1}{2a_{G_i}} + \frac{1}{2a_{G_j}} \right)^{-1} H_{lG_j} \end{aligned} \quad (23)$$

and for Region II

$$\frac{dF_l}{dQ_{z_k}} = H_{lG_j} \quad (24)$$

Since the nodes are aggregated based on nodal pricing only, it is possible that in the example above of Figure 5,  $\frac{dF_l}{dQ_{z_k}}$  varies considerably between Region I and Region II. Compared to nodal pricing method, the suboptimality of the solution to the optimization problem in Eq. (15) is mainly due to the aggregation step in which the sensitivity of flow of lines along congestion interfaces on zonal injection varies with respect to the region of operation as well as aggregate sensitivity within the region. With this type of aggregation, it is not always possible to choose the nodal injection point that is most effective in reducing flows on congested lines as in nodal pricing method. This can result in a large deviation in optimization

solution from the operating condition derived by employing the nodal pricing method. In the subsequent section we use another type of aggregation method where the nodes are grouped into zones based on this sensitivity factors in an attempt to reduce the degree of suboptimality.

### B. Congestion-Cluster Pricing Method

In congestion-cluster pricing method the nodes are aggregated into zones based on their relative impacts of injection on congested transmission lines. The key to the method, therefore lies on the ability to accurately measure these relative impacts, referred to as sensitivity in the previous section. For the implementation of this method the novel approach recently proposed in [14] is used to compute the sensitivity measures.

Similar to zonal pricing method, the system operator first receives the marginal supply bids from suppliers. Using these bids, the optimization problem given in Eq. (1) is solved to locate potentially congested transmission lines.

Once the potentially congested transmission lines are located, the system operator computes so-called *congestion distribution factors* (CDF) to identify the group of nodes which have similar effects on those transmission lines. This grouping is referred to as zonal aggregation into congestion clusters. We briefly describe the computation of CDF for completeness. The details of the approach can be found in [14].

CDFs are derived from distribution factors. First, distribution factors in usual sense are computed twice with respect to two different slack bus locations within the same system for transmission line of interest, i.e.  $\{D_m^{(i,j)}\}$  and  $\{D_n^{(i,j)}\}$  where bus  $n$  is used as the slack bus for the first computation, and bus  $m$  is for the second. Then, the difference between these two sets of distribution factors,  $\beta_{m,n}^{(i,j)}$ , is the result of having two slack buses in different location. Defining the difference as

$$\beta_{m,n}^{(i,j)}\{1\} = \{D_m^{(i,j)}\} - \{D_n^{(i,j)}\} \quad (25)$$

where  $\{1\}$  is the vector of all ones,  $\beta_{m,n}^{(i,j)}$ , can be expressed as [14]

$$\beta_{m,n}^{(i,j)} = D_m^{(i,j)}(n) = -D_n^{(i,j)}(m) \quad (26)$$

where  $D_m^{(i,j)}(n)$  denotes the  $n$ th element of the vector  $\{D_m^{(i,j)}\}$ .

Define the shift vector,  $\phi$  as

$$\phi^{i,j} = -\frac{D_m^{(i,j)}(i) + D_m^{(i,j)}(j)}{2} \quad (27)$$

for given distribution factors,  $\{D_m^{(i,j)}\}$  with respect to the slack bus,  $m$ . Then, we can subtract out the locational effect of slack bus from distribution factors by adding the sum of shift vector elements to the given distribution factors. The resulting vectors are what is defined as CDF,  $\{D^{(i,j)}\}$ :

$$\{D^{(i,j)}\} = \{D_m^{(i,j)}\} + \phi^{(i,j)}\{1\} \quad (28)$$

The magnitude of resulting CDF defines the sensitivity of the flow in transmission line of interest on a transaction; this formulation ensures that sensitivity of flows on the line of interest with respect to a bus injection decreases monotonically as the electrical distance between the line and the bus increases. The sign denotes if the transaction will increase or relieve the congestion.

Once nodes are aggregated into congestion clusters, the system operator then formulates the new generation cost function associated with each clusters. This yields the similar results to Eq. (14). For the rest of the steps, it follows the result directly from the zonal pricing method described in the previous section including solving optimization problem in (15) subject to constraints given in Eqs. (16), (17) and (18).

The computation of  $H_{lz_i}$  also follows the same approach starting from Eq. (19) to Eq. (22). However, the congestion-cluster pricing method differs from zonal pricing method in that due to the aggregation criteria used for clustering, the resulting  $H_{lz_i}$  yields a consistent value between various regions of operating conditions. That is to say,  $\frac{dF_l}{dQ_{z_j}}$  does not vary much regardless of the value of  $\frac{\partial Q_{G_i}}{\partial Q_{z_j}}$  since  $\frac{dF_l}{dQ_{G_i}} \approx \frac{dF_l}{dQ_{G_k}}$  if  $G_i \in z_j$  and  $G_k \in z_j$ .

#### IV. EXAMPLE

In this section an illustration is given for implementing the two cluster-based pricing methods described in the paper: the zonal pricing method and the congestion-cluster pricing method, through a simple 9-bus system example shown in Figure 6. We are mainly interested in the transfer constraint of 1MW on transmission line 3-7.

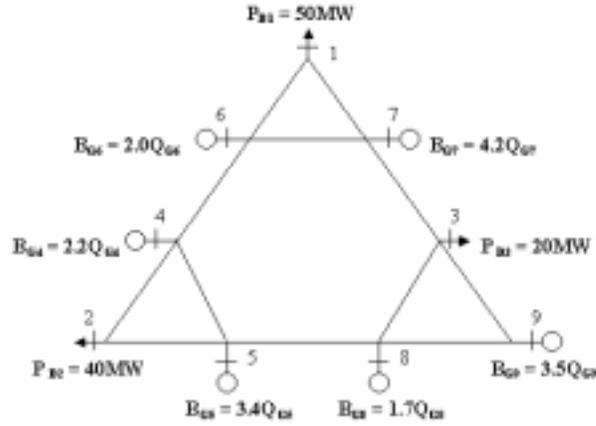


Fig. 6. One-line Diagram of 9-bus System

### A. Zonal Pricing Method

Using the bids from generators and given inelastic demand we solve the optimization problem given in (1) and calculate spot prices using (11). The results are shown in Figure 7. The different prices at each node reflect the different locational effect of each nodal injection

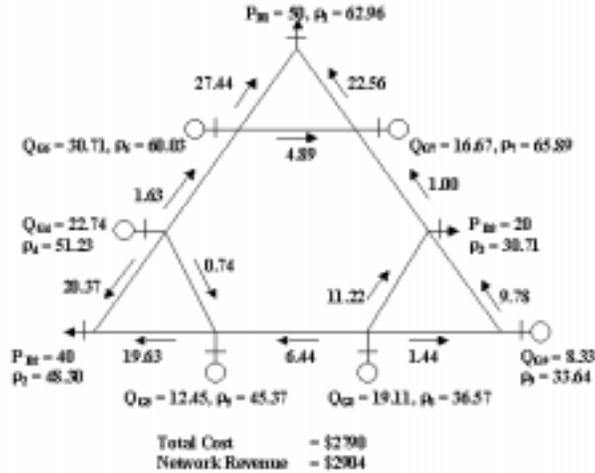


Fig. 7. Optimal Dispatch using the Nodal Pricing Method

on the congested line 3-7.

After solving the OPF, we form 3 zones by grouping nodes with similar nodal prices. The resulting zone 1 consists of nodes 1, 6 and 7; zone 2 consists of nodes 2, 4 and 5; and zone 3 consists of nodes 3, 8 and 9.

The bids from generators in each zone are then aggregated using procedures described in Eq. (19). The H-matrix components  $H_{lz}$  for the zones are calculated from Eq. (20). By

solving the optimization problem given in (15) we can compute the zonal prices. The results are shown in Figure 8

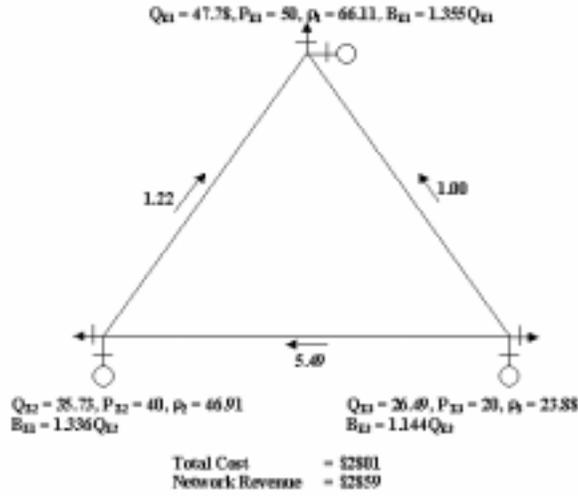


Fig. 8. Optimal Dispatch using the Zonal Pricing Method

To verify that the limit on line 3-7 is not violated, we perform the segregation of the 3 zonal dispatches into the 6 actual generators corresponding to the original 9-bus system. This is done by using Eq. (22) and solving the DC power flow given by (5). The resulting dispatch and line flows are shown in Figure 9.

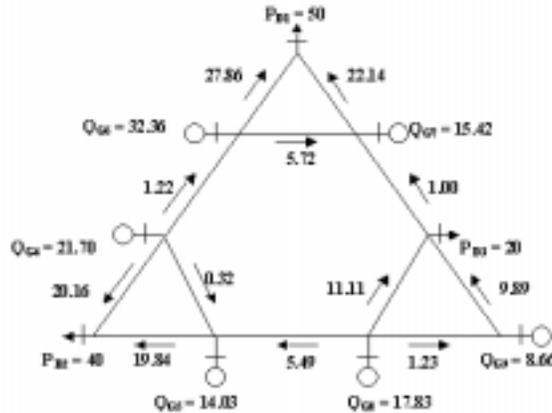


Fig. 9. The Resulting Flows Applying the Zonal Pricing Method

### B. Congestion-Cluster Pricing Method

In implementing the congestion-cluster pricing method we first calculate the congestion distribution factors using Eq. (28). The computed factors are presented in Figure 10.

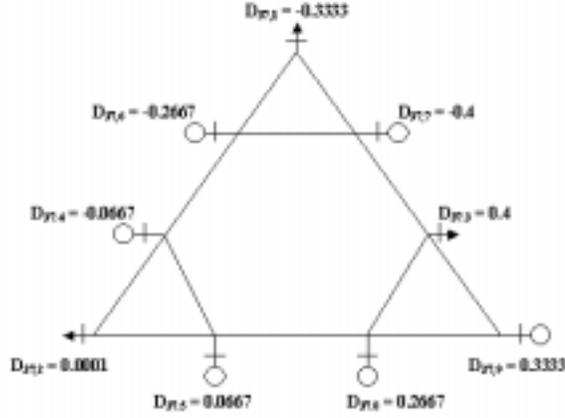


Fig. 10. The Congestion Distribution Factors for Congestion on Line 3

Based on the congestion distribution factors we form 4 clusters; nodes with similar CDF values are grouped into a cluster. Cluster 1 consists of nodes 1, 6 and 7; cluster 2 consists of node 4 only; cluster 3 consists of nodes 2 and 5; and cluster 4 consists of nodes 3, 8 and 9.

We then proceed in the manner similar to the zonal pricing method to derive the cluster-wide optimal dispatch and prices by solving (15) using (19) and (20). The results are shown in Figure 11.

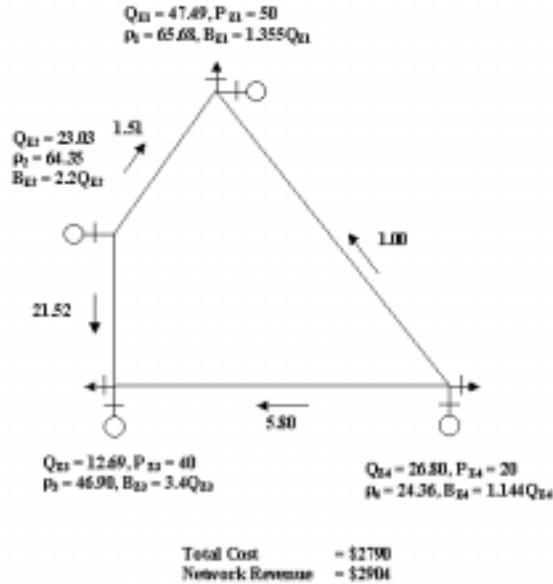


Fig. 11. Optimal Dispatch using the Congestion-cluster Pricing Method

To verify that the limit on line 3-7 is not violated, we segregate the 3 cluster-wide dispatches into the 6 actual generators corresponding to the original 9-bus system by employing Eq. (22), and solve the DC power flow given in Eq. (5). The resulting dispatch and line flows

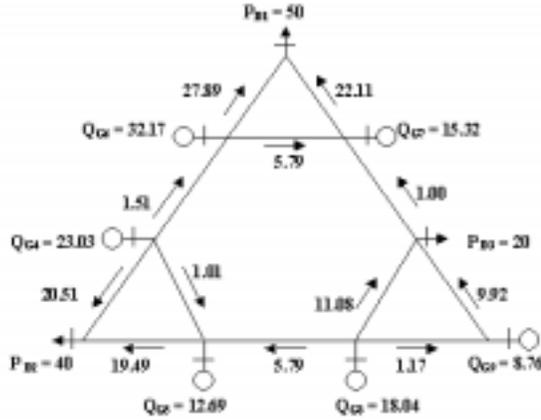


Fig. 12. The Resulting Flows Applying the Congestion-cluster Pricing Method

are shown in Figure 12.

## V. CONCLUSION

In this paper the systematic procedures are presented for implementing cluster-based congestion management systems. We derive the equations needed for the implementation which requires two steps: (1) aggregation of individual nodes into clusters and (2) computation of cluster-wide prices.

The zonal pricing method and the congestion-cluster pricing method are described in details as the examples of cluster-based congestion management systems. The difficulty in aggregation step is demonstrated to show that the suboptimality related to cluster-based congestion management systems compared to the nodal pricing method. A high degree of operational insights is required to reduce the suboptimality as well as to prevent the congestion within each cluster.

This suboptimality in the short term efficiency relative to the nodal pricing method is believed to be more than compensated through the increase in the long term efficiency as through the transparent information of cluster boundaries, the market participants are more aware how their activities affect the congestion in the transmission system. Many market related functions including investment directly depend on the knowledge of this congestion effect.[8]

There are several issues to be investigated for extending the result presented in the paper for practical application on a large scale systems.[12] The most pronounced problem is related to

multiple line congestion over a period of time where the uncertainty is significant. Some work is underway, and the preliminary result shows that the congestion-cluster pricing method is more effective in reducing the suboptimality without losing the transparent information structure. This result and other related work are expected to be reported in the subsequent papers.

## REFERENCES

- [1] Alaywan, Z., "Facilitating the Congestion Management Market in California", California Independent System Operator, April, 1999.
- [2] Bertsekas, D., "Nonlinear Programming", Athena Scientific, 1999.
- [3] Energy Information Administration, *Electric Power Annual* Volume II, DOE/EIA-0348(96/2), Washington, DC, December 1997, pp. 13-14.
- [4] Einhorn, M., Siddiqi, R., "Electricity Transmission Pricing and Technology", Kluwer Academic Publishers, 1996
- [5] Gedra, T., "On Transmission Congestion and Pricing", IEEE Transactions on Power Systems, Vol. 14, No 1, pp. 241-248, February 1999.
- [6] Hogan, W., "Contract Networks for Electric Power Transmission: Technical Reference", Harvard University, September 1990.
- [7] Huneault, M., Galiana, F., Gross, G., "A Review of Restructuring in the Electricity Business", 13th PSCC in Trondheim, June 28-July 2, 1999.
- [8] Ilic, M., Galiana, F., Fink, L., "Power systems Restructuring. Engineering and Economics", Kluwer Academic Publishers. 1998
- [9] Ilic, M., Hyman, L., Allen, E., Younes, Z., "Transmission Scarcity: Who pays? The Electricity Journal, July 1997, pp. 38-49.
- [10] Leotard, J-P., Ilic, M.D., "On the Objectives of Transmission Pricing Under Open Access", IEEE Winter Meeting, January 1999, pp 476-483.
- [11] Oren, S., Spiller, P., Varaiya, P., Wu, F., "Nodal Prices and Transmission Rights: a Critical Appraisal", University of California, Berkeley, December, 1994.
- [12] Perez Arriaga, I.J., Rubio F.J., Puerta J.F., Arceluz, J.M., "Marginal Pricing of Transmission Services: An Analysis of Cost Recovery", Transactions on Power Systems, Vol. 10, No. 1, February 1995.
- [13] Schweppe, F., Caramanis, M., Tabors, R., Bohn, R., "Spot Pricing of Electricity", Kluwer Academics, 1988.
- [14] Yu, C-N., Ilic, M.D., "Congestion Cluster-Based Markets for Transmission Management", IEEE Winter Meeting, January 1999, pp 821-832.