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Pricing and hedging in carbon emissions markets

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Abstract

We propose a model for trading in emission allowances in the EU Emission Trading Scheme (ETS). Exploiting an arbitrage relationship we derive the spot prices of carbon allowances given a forward contract whose price is exogenous to the model. The modeling is done under the assumption of *no banking* of carbon allowances (which is valid during the Phase I of Kyoto protocol), however, we also discuss how the model can be extended when banking of permits is available. We employ results from filtering theory to derive the spot prices of permits and suggest hedging formulas using a local risk minimisation approach. We also consider the effect of intermediate announcements regarding the net position of the ETS zone on the prices and show that the jumps in the prices can be attributed to information release on the net position of the zone. We also provide a brief numerical simulation for the price processes of carbon allowances using our model to show the resemblance to the actual data.

Keywords: CO_2 emission allowances, EU ETS, incomplete information, stochastic filtering, minimal martingale measure.

1 Introduction

Global warming and its dangerous consequences have gained increased public attention in recent years. There is now broad scientific consensus that greenhouse gas emissions related to human activities are responsible for the increase in atmospheric temperatures recorded since the middle of the nineteenth century. Sustained economic growth since the industrial revolution has gone hand in hand with an increased burning of fossil fuels such as coal, oil and gas in a chemical process that releases carbon dioxide (CO_2), one of the most abundant greenhouse gases. Various scientific studies have pointed at the severe economic burden of global warming for future generations if rising concentrations of CO_2 and other greenhouse gases are not curbed within the next twenty to thirty years, see [6] for a recent account.

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The Kyoto protocol was opened for signature at the 1997 conference in Kyoto, Japan and assigns mandatory emission limits for greenhouse gases to signatory nations. These nations must reduce their emissions for carbon dioxide and five other gases in 2008-2012 by 5% with respect to 1990 levels in order to comply with the protocol. Signatory nations that do not meet these targets will be penalized, but emission rights, called 'allowances', may otherwise be traded bilaterally in a process referred to as 'carbon trading'.

The European Commission launched the European Climate Change Programme (ECCP) in June 2000 with the objective to identify, develop and implement the essential elements of an EU strategy to implement the Kyoto Protocol. All 25 EU countries simultaneously ratified the Kyoto Protocol on 31 May 2002. The European Union Emission Trading Scheme (EU ETS) is a significant part of the ECCP and currently constitutes the largest emissions trading scheme in the world. It is widely regarded as a trial phase for the eventual Kyoto period (2008-2012) during which carbon emission allowances will become a traded commodity on an even larger, global scale. To participate in the ETS, EU member states must first submit a National Allocation Plan (NAP) for approval to the European Commission. Selected carbon intensive installations such as steel manufacturers, power stations of above 20 MW capacity, cement factories, etc. receive free emission allowances under the terms of this NAP, enabling them to emit greenhouse gases up to the assigned tonnage.

Installations can bilaterally trade emission allowances under the EU ETS, in order to offset any excess or shortage of carbon emission permits compared to the NAP issuance. About 12.000 installations within the Union are covered by the EU ETS in a first phase (2005-2007), representing almost 50% of total carbon emissions. The EU ETS enables selected industries to reduce carbon emissions in a cost effective manner, i.e. carbon emissions come at a cost but installations can opt for either reducing actual carbon emissions or buying additional allowances, for instance in case upgrading of the installation would turn out more expensive. The NAP only imposes a cap on the total actual carbon emissions per member state.

Actual trading in EU ETS emission allowances began January 1st, 2005. By the end of the same year, almost 400 million tonnes of carbon equivalent had been traded, representing a turnover in excess of EUR 7 billion. The impact of the release of sensitive information regarding the ETS net position in carbon emission allowances can be dramatic, as was illustrated in April 2006. First phase EU ETS carbon, in the form of the allowance expiring in December 2007, written Dec-07, had reached EUR 30 per tonne at their high in April 2006. Prices subsequently plumetted to below EUR 10 per tonne in a few days beginning May 2006 (see Figure 1) after EU figures on actual 2005 emission levels suggested emission caps to selected industries had been too generous to have a significant impact on emission practice. Emission caps for the second phase (2008-2012) are currently under review because of this apparent generosity of NAP levels in the first phase.

Every installation included in the EU ETS has to surrender carbon allowances at the end of April of every calendar year in order to cover its emissions during the preceding year. In case the installation is not able to surrender enough allowances a penalty of EUR 100 (EUR 40 for Phase I) for each tonne of excess emission is to be paid. Payment of the excess emission penalty, however, does not release the installation from the obligation to surrender the number of allowances equal to those excess emissions. The company is required to provide these allowances when surrendering the carbon allowances in relation to the following calendar year. The essential difference between Phase I and II lies in the fact that the banking of allowances is not permitted during Phase I and, thus, the allowances expires at their stated maturity, while the allowances issued during Phase II can be banked by the installations for future use. The mechanism described above provides a direct link between the spot and forward prices of carbon allowances. To see this consider, e.g., the so-called Dec-07 and Dec-08 contracts that had been traded in the EU ETS during 2007. Dec-07 contract expires at the end of December 2007 and can be used to cover emissions during 2007. On the other hand Dec-08 contract has maturity December 2008 and can only be used to cover emissions during the calendar year of 2008. Since these two contracts are traded during Phase I in which banking is not allowed Dec-07 contracts cannot be used to cover emissions in 2008. Therefore, the prices of Dec-07 contracts can be viewed as the spot price for carbon allowances during 2007 and the Dec-08 contracts can be viewed as a forward contract. Prices of Dec-07 contracts at the end Dec-07 will be zero if the EU zone is net long EU ETS allowances in 2007. Indeed, since the zone is net long there are some firms holding carbon allowances that they do not need. We may assume there are at least two such firms since the number of installations in the scheme is large enough. The competition among these firms will drive the price of the Dec-07 allowances to 0. On the other hand, if the zone is net short then Dec-07 contracts will not be worthless and their price will be equal to the price of Dec-08 contract plus the penalty due to the aforementioned regulations on the surrendering of the allowances. the Industries can then opt for borrowing their short ETS position into the next phase at a fixed cost of EUR 40 per tonne.

A similar arbitrage relationship between the spot and forward prices of carbon exists in Phase II. However, the situation is a little more complicated due to banking. In particular the price of the spot will not be zero even if the zone is net long since the current allowances can be used later to cover emissions. We discuss how the model can be extended to incorporate banking of the permits in Section 7. Banking is also permitted between phase II and phase III (2013-2017), but as the mechanism beyond the 2012 Kyoto deadline is still largely unknown, we will not address the consequences of this additional optionality for now.

In this paper, we provide a framework to study the spot prices of carbon allowances traded in EU ETS. Using the above no-arbitrage principle between the *spot* and *forward* prices of carbon allowances we shall derive the pricing formulas for the carbon allowances by assuming an exogenous price process for the forward contract. We study the pricing using a local risk minimising criteria under two settings where the market's net position is common knowledge and is not. This approach also enables us to come up with the optimal hedging strategy for the spot contract. In this incomplete information setting we use techniques from stochastic filtering theory in order to model the market's estimate for the net position. Moreover, our approach enables us to calculate the probability of net long position.

The relationship between the probability of market being long and the prices of EUA contracts has also been observed in [1]. Their observation is based on their intuition and

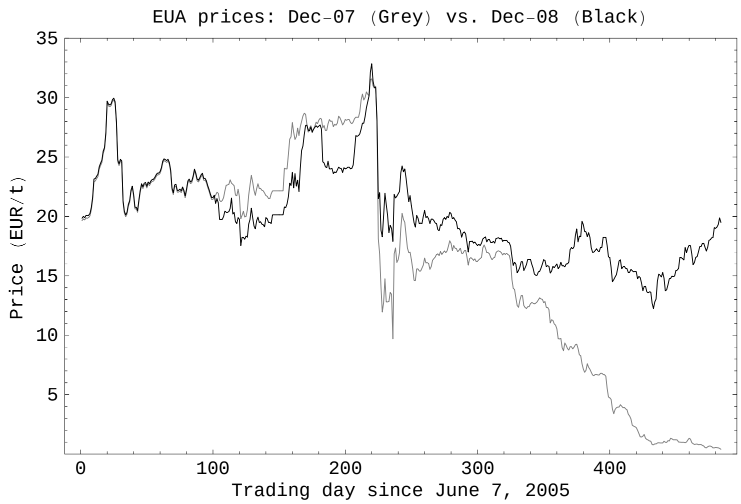

Figure 1: EUA price history between June 7th, 2005 and May 5th, 2007

they do not intend to do a rigorous mathematical analysis. Moreover, to the best of our knowledge, an in-depth quantitative analysis of the pricing and hedging issues in EU ETS is yet to be done. Paolella and Taschini [8] makes an econometric analysis of EU ETS market. Seifert, et al. [11] and Fehr and Hinz [2] study the spot carbon prices in equilibrium. Seifert, et al. avoid the natural complications of an equilibrium analysis by assuming every market participant risk-neutral or the existence of a representative agent with a logarithmic utility, thereby reducing the problem to the problem of a central planner who aims to maximize the total profit of all agents. Fehr and Hinz study an equilibrium among N market participants. Although the setting is more realistic, the model only gives a characterization of spot prices but does not produce explicit solutions. In this paper we avoid the equilibrium approach to spot prices but use aforementioned arbitrage relationship to come up with an explicit semimartingale represenation for the carbon spot prices.

The outline of the paper is as follows. Section 2 introduces the underlying model for the pricing of carbon credits. Section 3 studies the pricing and hedging of carbon credits under the complete information on the market's net position while Section 4 studies the same problem under, more realistic, incomplete information setting. Section 5 discusses the effects of intermediate announcements of the net position over the spot prices. Section 6 presents a numerical study and Section 7 concludes. In the appendix we provide a glossary of terms and abbreviations used in the paper for the convenience of the reader.

2 Model

We start with a filtered probability space, $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, \mathbb{P})$, satisfying usual conditions. All stochastic processes in what follows will be defined on and adapted to this space. We consider a market for the trading of the EUAs (carbon allowances traded in the EU ETS scheme) as described in the introduction. For simplicity, we assume there are two EUAs traded in the market: EUA for the current year, denoted with EUA0, and EUA for the next year, denoted with EUA1. An example could be the market for Dec-07 and Dec-08 contracts traded in EU ETS 2007. We suppose the price process, S, for the EUA1 contract is a continuous process satisfying

$$dS_t = S_t\mu(t, S_t, \theta_t)dt + S_t\sigma(t, S_t, \theta_t)dW_t, \tag{2.1}$$

with $S_0 = s$. Here, θ is a Markov chain, which we shall describe shortly, modelling the net position of the market, and W is a Brownian motion independent of θ. Thus, we assume (θ, S) is a vector Markov process. In order to have transparent results we simplify the modelling of S by assuming

$$\sigma(t, s, x) = \sigma \qquad \text{and} \qquad \mu(t, s, x) = \mu + \alpha\theta, \tag{2.2}$$

for some constants σ, μ and α, for all t, s, and x.

As seen, the drift term depends on the position of the market. The process θ is supposed to be a càdlàg Markov chain in continuous time taking values in $E := \{-1, 1\}$. The modelling idea is that $\theta_t = 1$ (resp. $\theta_t = -1$) corresponds to market being long (resp. short) at time t. The heuristics behind this choice relate to the fact that interest in the current year EUA contract decreases as the zone proves long allowances, implying that market participants 'roll' their position one year forward which, in its turn, affects the price for the allowance for delivery in the next year. Therefore, the corresponding drift term when market is long is $\mu + \alpha$ while the drift equals $\mu - \alpha$ when the market is short. The assumption that θ takes only two values is for simplicity and our theory can be extended to the case when E is any finite set, although some extra effort in calculations will be needed. When E has more than two elements, the states of the Markov chain can be considered as an indicator of how long or short the overall position of the market is.

As explained in Introduction, in the case of excess emissions, the payment of the penalty does not remove the obligation to deliver permits, which means undelivered permits still have to be handed in the next calendar year. Hence, in the case of permit shortage, one would expect an upwards shift in the demand for the EUA1 contracts. In view of this observation we typically expect α to be negative.

We suppose the Markov chain θ stays in state i for an exponential amount of time with parameter $\lambda(i)$. More precisely, if R is a 2×2 matrix of transition probabilities with entries $R_t(i, j) := \mathbb{P}(\theta_t = j | \theta_0 = i)$ and Q is the generator matrix defined by

$$Q = \begin{pmatrix} -\lambda(1) & \lambda(1) \\ \lambda(2) & -\lambda(2) \end{pmatrix},$$

then the transition probabilities solves the forward equation

$$R_t' = R_t Q, \qquad R_0 = I, \tag{2.3}$$

where I is the identity matrix and $R_t'(i,j) = \frac{d}{dt} R_t(i,j)$. We denote the initial distribution for θ with p. The following result is well-known (see e.g. [4])

Lemma 2.1 *Let θ be the right-continuous Markov chain taking values in E with the initial distribution p and the generator matrix Q. Then,*

$$\theta_t = \theta_0 - 2\int_0^t \theta_s \lambda(\theta_s) ds + N_t, \tag{2.4}$$

for each t, where N is a martingale. Moreover, the decomposition $\theta = \theta_0 + N + A$ where N is a martingale and A is a predictable process with $N_0 = A_0 = 0$ is unique.

The following assumption is to simplify the computations and the exposition. We stress here that our approach still works without the next assumption.

Assumption 2.1 $\lambda(1) = \lambda(2) = \lambda$.

Now we turn to the pricing of EUA0 contracts. Under the assumption of no banking of permits these contracts will be worthless if market ends up long at their respective expiry date, which we denote with T. If the market is instead short at time T, then these contracts can be turned into an EUA1 contract by paying a penalty K. Letting P denote the price process of EUA0 contracts, this implies we have the following relation between S and P at time T

$$P_T = \begin{cases} S_T + K, & \text{if } \theta_T \leq 0; \\ 0, & \text{otherwise,} \end{cases} \tag{2.5}$$

so that EUA0 can be considered as an option on EUA1. In the next two sections we will discuss the *local risk minimisation* approach to the pricing and hedging in EU ETS market.

3 Pricing and hedging under complete information

The model introduced above for the prices of carbon allowances is typically incomplete. This is because the price process S depends on two sources of uncertainity, W and θ, while θ is not tradable. As we are in an incomplete setting, there is usually an interval of arbitrage-free prices, and depending on the chosen approach one could come up with different prices and hedging strategies for the same derivative. In this paper we will use the so-called *local-risk minimisation* approach and for background reading we refer to Schweizer [10], Fölmer and Schweizer [3], and Monat and Stricker [7]. We suppose the traders have access to full information about the market so that they have the filtration $\mathbb{F} := (\mathcal{F}_t)_{0 \leq t \leq T}$. Following [3] we define the *optimal* hedging strategy for a given contingent claim as follows.

Definition 3.1 *Let $C \in L^2(\Omega, \mathcal{F}_T, \mathbb{P})$ be a contingent claim. A predictable trading strategy ξ^C is said to be optimal if there exists a square integrable $\mathbb{F}$-martingale, L^C, orthogonal to W such that*

$$C = c + \int_0^T \xi_t dS_t + L_T^C. \tag{3.6}$$

A second look at the decomposition in (3.6) reveals that $\int_0^T \xi_t dS_t$ corresponds to the part of the risk that is hedgable. L_T^C on the other hand corresponds to the intrinsic risk associated with the contingent claim that is not hedgable. Existence of (3.6) is intimately linked to the so-called *minimal* martingale measure.

Definition 3.2 *Let X be a continuous semimartingale with the canonical decomposition $X = X_0 + M + A$ with M a martingale and A is adapted, continuous and of finite variation. A probability measure $\widehat{\mathbb{P}} \sim \mathbb{P}$ is called* minimal martingale measure *if X follows a martingale under $\widehat{\mathbb{P}}$, $\widehat{\mathbb{P}} = \mathbb{P}$ on $\mathcal{F}_0$ and any square integrable martingale orthogonal to M remains a martingale under $\widehat{\mathbb{P}}$.*

The minimal martingale measure is uniquely determined (see, e.g., [3]) and in our case is defined by

$$\frac{d\widehat{\mathbb{P}}}{d\mathbb{P}} = \exp\left(-\int_0^T \frac{\mu + \alpha\theta_s}{\sigma} dW_s - \frac{1}{2}\int_0^T \left(\frac{\mu + \alpha\theta_s}{\sigma}\right)^2 ds\right). \tag{3.7}$$

Note that

$$G_t := \exp\left(-\int_0^t \frac{\mu + \alpha\theta_s}{\sigma} dW_s - \frac{1}{2}\int_0^t \left(\frac{\mu + \alpha\theta_s}{\sigma}\right)^2 ds\right)$$

defines a square integrable martingale.

The following theorem is from [3].

Theorem 3.1 (Theorem 3.14 in [3]) *Let $C \in L^2(\Omega, \mathcal{F}_T, \mathbb{P})$ and let $\widehat{\mathbb{P}}$ be the unique minimal martingale measure for S given by (3.7). Then there exists a unique ξ^C such that*

$$V_t = \widehat{\mathbb{E}}[C] + \int_0^t \xi_s^C dS_s + L_t^C,$$

where L^C is an $\mathbb{F}$-martingale orthogonal to W and $V_t := \widehat{\mathbb{E}}[C|\mathcal{F}_t]$.

In view of the above theorem we conclude that ξ^C defined by the Radon-Nikodym derivative

$$\xi_t^C := \frac{d\langle V, S\rangle}{d\langle S\rangle}$$

is the optimal strategy in the sense of Definition 3.1. The following definition gives the price of a contingent claim under complete information.

Definition 3.3 *Let $C \in L^2(\Omega, \mathcal{F}_T, \mathbb{P})$ be a contingent claim and let $\widehat{\mathbb{P}}$ be the unique minimal martingale measure for S given by (3.7). The fair price P_t^C at time t for C is given by*

$$P_t^C := \widehat{\mathbb{E}}[C|\mathcal{F}_t].$$

For the problem under consideration $C = (S_T+K)\mathbf{1}_{\{\theta_T=-1\}} = \frac{S_T+K}{2}(1-\theta_T)$. In the remaining of this section we calculate the price and hedging strategy for EUA0 contracts. In view of Lemma 2.1, $\theta = \theta_0+N+A$ where N is a martingale and A is a predictable process defined by $A_t = -2\lambda\int_0^t \theta_s ds$. Since θ and W are independent, it follows that N and W are orthogonal martingales under $\mathbb{P}$; thus, N remains a martingale under $\widehat{\mathbb{P}}$ and is orthogonal to S. Since S is square integrable under $\widehat{\mathbb{P}}$,

$$\widehat{\mathbb{E}}\left[\int_0^T A_s^2 d\langle S\rangle_s\right] < \infty,$$

so that $\int_0^\cdot A_s dS_{s\wedge T}$ is a square integrable martingale. Therefore, letting $\widehat{\mathbb{E}}_t[\cdot]$ denote $\widehat{\mathbb{E}}[\cdot|\mathcal{F}_t]$,

$$\begin{aligned} P_t &= \widehat{\mathbb{E}}_t\left[\frac{S_T+K}{2}(1-\theta_0-N_T) - \frac{S_T+K}{2}A_T\right] \\ &= \frac{S_t+K}{2}(1-\theta_0-N_t) - \widehat{\mathbb{E}}_t\left[\frac{S_T+K}{2}A_T\right]. \end{aligned} \tag{3.8}$$

The only term that needs to be calculated is the expectation in (3.8). For this we will use the explicit form for A.

Proposition 3.1 *Let*

$$M_t := \theta_t \exp(-2\lambda(T-t)), \qquad t \in [0,T].$$

Then M is a $\widehat{\mathbb{P}}$-martingale and

$$\widehat{\mathbb{E}}_t\left[\frac{S_T+K}{2}A_T\right] = \theta_t\frac{S_t+K}{2}\exp(-2\lambda(T-t)) - (\theta_0+N_t)\frac{S_t+K}{2}.$$

Proof. Let

$$Y_t = \theta_t\frac{S_t+K}{2}\exp(-2\lambda(T-t)) - (\theta_0+N_t)\frac{S_t+K}{2}.$$

Since $Y_T = \frac{S_T+K}{2}A_T$ it suffices to show Y is $\widehat{\mathbb{P}}$-martingale. Since N is a $\widehat{\mathbb{P}}$-martingale orthogonal to S it follows that $(\theta_0+N)(S+K)/2$ is a $\widehat{\mathbb{P}}$-martingale. Moreover,

$$\begin{aligned} \theta_t\exp(-2\lambda(T-t)) &= \theta_0\exp(-2\lambda T) + \int_0^t \exp(-2\lambda(T-s)d\theta_s + \int_0^t 2\lambda\theta_s\exp(-2\lambda(T-s))ds \\ &= \theta_0\exp(-2\lambda T) + \int_0^t \exp(-2\lambda(T-s)dN_s \end{aligned}$$

since $dA_t = -2\lambda\theta_t dt$. This implies $(M_t)_{0\le t\le T}$ is a martingale orthogonal to S so that $(\theta_t\frac{S_t+K}{2}\exp(-2\lambda(T-t)))_{0\le t\le T}$ is a martingale, too. ■

Summing up the above calculations we have the following result:

Theorem 3.2 *The fair price for EUA0 contracts is given by*

$$P_t = (S_t + K)\frac{1 - \theta_t \exp(-2\lambda(T-t))}{2}.$$

The optimal hedging strategy, ξ^0 associated with EUA0 contracts is given by

$$\xi_t^0 := \frac{1 - M_t}{2},$$

for each $t \in [0, T]$.

Proof. Expression for the price follows from (3.8) and Proposition 3.1. To find the hedging strategy it suffices to find the integral representation for P, which equals

$$P_t = (S_0 + K)(1 - \theta_0 \exp(-2\lambda T)/2) + \int_0^t \frac{1 - M_s}{2} dS_s - \int_0^t (S_s + K) dM_s,$$

for each $t \in [0, T]$. ■

In other words, part of the risk at time t, corresponding the term $\int_0^t \frac{1-M_s}{2} dS_s$, associated to the claim C_T can be hedged if one follows the *locally-risk minimising strategy* which consist of holding $(1 - M)/2$ shares of the traded underlying, whose price process is given by S.

4 Pricing under incomplete information

The EU ETS market participants typically do not observe θ continuosly. In this section we study the pricing of EUA0 contracts under incomplete information. We suppose the only information available to the market is the usual right-continous and complete augmentation of S, denoted with $\mathcal{F}^S$ and the one-time announcement of the true value of θ at time T. If $\mathcal{G}$ denotes the filtration modelling the information structure of the market, then

$$\mathcal{G}_t = \begin{cases} \mathcal{F}_t^S, & \text{for } t < T; \\ \mathcal{F}_T^S \vee \sigma(\theta_T), & \text{for } t = T. \end{cases}$$

Let $\overline{\theta}$ denote the optional projection of θ to $\mathcal{F}^S$ which gives $\overline{\theta}_t = \mathbb{E}[\theta_t | \mathcal{F}_t^S]$, for each $t \geq 0$. We now apply the aforementioned local-risk minimisation approach to the pricing and hedging of EUA0 under incomplete information, i.e. when the available information is modelled by $\mathcal{G}$.

Theorem 4.1 *Define $\overline{W}$ by*

$$\overline{W}_t = \int_0^t \frac{dS_s - (\mu + \alpha\overline{\theta}_s)S_s ds}{\sigma S_s},$$

and Z by $Z_t = \mathbf{1}_{[t=T]}(\theta_T - \overline{\theta}_T)$ for each $t \geq 0$. Then, $\overline{W}$ and Z are orthogonal $\mathcal{G}$-martingales. Moreover, $\overline{W}$ is a $\mathcal{G}$-Brownian motion.

PROOF. It follows from standard filtering theory, see, e.g., [5], that $\overline{W}$ is an $\mathcal{F}^S$-Brownian motion. Observe that $\overline{W}$ stays a martingale when $\mathcal{F}^S$ is enlarged with $\sigma(\theta_T)$ at time T. This is simply because $\mathbb{E}[\overline{W}_T|\mathcal{G}_t] = \mathbb{E}[\overline{W}_T|\mathcal{F}^S_t] = \overline{W}_t$, for each $t < T$. This shows that $\overline{W}$ is a $\mathcal{G}$-Brownian motion as well by Lévy's characterisation of Brownian motion. To show Z is a $\mathcal{G}$-martingale observe that, for $t < T$, $\mathbb{E}[Z_T|\mathcal{G}_t] = \mathbb{E}[Z_T|\mathcal{F}^S_t] = \mathbb{E}[\mathbb{E}[Z_T|\mathcal{F}^S_T]|\mathcal{F}_t] = 0 = Z_t$. Z and $\overline{W}$ are orthogonal since $[\overline{W}, Z] = 0$. ■

Using standard filtering theory (see Theorem 8.1 in [5]) and the fact that $\theta^2 = 1$, we have

$$d\overline{\theta}_t = -2\lambda\overline{\theta}_t dt + \frac{\alpha}{\sigma}(1 - \overline{\theta}_t^{\,2})d\overline{W}_t, \tag{4.9}$$

with $\overline{\theta}_0 = 2p - 1$. Note that under the smaller filtration, $\mathcal{G}$, $\widehat{\mathbb{P}}$ will no longer be the minimal martingale measure for S. However, there is still a unique minimal martingale measure, denoted with $\mathbb{P}^*$, associated with S with respect to $\mathcal{G}$.

Definition 4.1 *The fair price of the EUA0 contracts at time t under incomplete information is defined to be*

$$\overline{P}_t := \mathbb{E}^*[(S_T + K)(1 - \theta_T)/2|\mathcal{G}_t],$$

where $\mathbb{E}^*$ is the expectation operator under $\mathbb{P}^*$.

Note that

$$\begin{aligned} P_t &= \mathbb{E}^*[(S_T + K)(1 - \theta_T)/2|\mathcal{G}_t] = \mathbb{E}^*[(S_T + K)(1 - \overline{\theta}_T)/2|\mathcal{G}_t] - \mathbb{E}^*[(S_T + K)Z_T/2|\mathcal{G}_t] \\ &= \mathbb{E}^*[(S_T + K)(1 - \overline{\theta}_T)/2|\mathcal{G}_t] - (S_t + K)\frac{Z_t}{2}. \end{aligned} \tag{4.10}$$

Theorem 4.2 *Under $\mathbb{P}^*$, $(S, \overline{\theta})$ is a vector Markov process. Moreover,*

$$dS_t = \sigma S_t dW^*_t, \qquad S_0 = s; \tag{4.11}$$

$$d\overline{\theta}_t = -\left(2\lambda\overline{\theta}_t + \frac{\alpha}{\sigma^2}(1 - \overline{\theta}_t^{\,2})(\mu + \alpha\overline{\theta}_t)\right)dt + \frac{\alpha}{\sigma}(1 - \overline{\theta}_t^{\,2})dW^*_t, \ \overline{\theta}_0 = 2p - 1, \tag{4.12}$$

where W^* is a $(\mathcal{G}, \mathbb{P}^*)$-Brownian motion and $p = \mathbb{P}[\theta_0 = 1]$.

PROOF. It suffices to show $(S, \overline{\theta})$ satisfies (4.11) and (4.12), which will in turn imply $(S, \overline{\theta})$ is Markov. That S satisfies (4.11) follows from the definition of $\mathbb{P}^*$. Then,

$$\begin{aligned} d\overline{\theta}_t &= -2\lambda\overline{\theta}_t dt + \frac{\alpha}{\sigma}(1 - \overline{\theta}_t^{\,2})d\overline{W}_t \\ &= -\left(2\lambda\overline{\theta}_t + \frac{\alpha}{\sigma^2}(1 - \overline{\theta}_t^{\,2})(\mu + \alpha\overline{\theta}_t)\right)dt + \frac{\alpha}{\sigma^2}\frac{1 - \overline{\theta}_t^{\,2}}{S_t}dS_t. \end{aligned}$$

■

Theorem 4.3 *Let $h : \mathbb{R}_+ \times \mathbb{R}_+ \times [-1,1] \mapsto \mathbb{R}$ be the solution to the boundary value problem*

$$\begin{aligned} h_t(t,x,y) + \mathcal{L}h(t,x,y) &= 0, \\ h(T,x,y) &= \frac{x+K}{2}(1-y), \end{aligned}$$

where

$$\begin{aligned} \mathcal{L}h(t,x,y) &:= \frac{1}{2}\sigma^2 x^2 h_{xx}(t,x,y) + \frac{1}{2}\frac{\alpha^2}{\sigma^2}(1-y^2)^2 h_{yy}(t,x,y) + \alpha x(1-y^2)h_{xy}(t,x,y) \\ &\quad - h_y(t,x,y)\left(2\lambda y + \frac{\alpha}{\sigma^2}(1-y^2)(\mu+\alpha y)\right). \end{aligned}$$

Then

$$\overline{P}_t = h(t,S_t,\overline{\theta}_t) - Z_t\frac{S_t+K}{2}.$$

Moreover, the optimal strategy under incomplete information associated to EUA0 contracts is given by the process $\overline{\xi} = (\overline{\xi}_t)_{0\le t\le T}$ where

$$\overline{\xi}_t := h_x(t,S_t,\overline{\theta}_t) + \frac{\alpha}{\sigma^2}\frac{1-\overline{\theta}_t^2}{S_t}h_y(t,S_t,\overline{\theta}_t),$$

for each $t \in [0,T]$.

Proof. Note that $h(T,S_T,\overline{\theta}_T) = \frac{S_T+K}{2}(1-\overline{\theta}_T)$. Thus, it remains to show $\big(h(t,S_t,\overline{\theta}_t)\big)_{0\le t\le T}$ is a $\mathcal{G}$-martingale. Using Itô's formula

$$\begin{aligned} h(t,S_t,\overline{\theta}_t) &= h(0,S_0,\overline{\theta}_0) + \int_0^t \left\{ h_x(s,S_s,\overline{\theta}_s) + \frac{\alpha}{\sigma^2}\frac{1-\overline{\theta}_s^2}{S_s}h_y(s,S_s,\overline{\theta}_s)\right\} dS_s \\ &\quad + \int_0^t h_t(s,S_s,\overline{\theta}_s) + \mathcal{L}h(s,S_s,\overline{\theta}_s)ds \\ &= h(0,S_0,\overline{\theta}_0) + \int_0^t \left\{ h_x(s,S_s,\overline{\theta}_s) + \frac{\alpha}{\sigma^2}\frac{1-\overline{\theta}_s^2}{S_s}h_y(s,S_s,\overline{\theta}_s)\right\} dS_s. \end{aligned}$$

Moreover, the integrand in the last integral is the optimal strategy. ■

The above methodology also enables us to price and hedge contingent claims on θ such as digital options. Suppose there exists a digital option which pays EUR 1 if the market is short at time T. If D is the fair price process of this digital option then $D_t = \mathbb{E}^*[(1-\theta_T)/2|\mathcal{G}_t]$ due to earlier arguments. Thus, $D_t = \mathbb{E}^*[(1-\overline{\theta}_T)/2|\mathcal{G}_t] - Z_t/2$. Proceeding along the similar lines for the pricing of EUA0 contracts we have the following result.

Theorem 4.4 *Consider a digital option which pays EUR 1 if the market is short at time T. The fair price process, D, of this option is given by*

$$D_t = h(t,\overline{\theta}_t) - \frac{Z_t}{2},$$

where h is the solution to

$$\begin{aligned} h_t(t,y) + \mathcal{L}h(t,y) &= 0, \\ h(T,y) &= \frac{1-y}{2}, \end{aligned}$$

and

$$\mathcal{L}h(t,y) := \frac{1}{2}\frac{\alpha^2}{\sigma^2}(1-y^2)^2 h_{yy}(t,x,y) - h_y(t,x,y)\left(2\lambda y + \frac{\alpha}{\sigma^2}(1-y^2)(\mu+\alpha y)\right).$$

Moreover, the optimal strategy under incomplete information associated to the digital option is given by $\xi^D = (\xi^D_t)_{0\le t\le T}$ where

$$\xi^D_t := \frac{\alpha}{\sigma^2}\frac{1-\overline{\theta}_t^2}{S_t} h_y(t,\overline{\theta}_t),$$

for each $t \in [0,T]$.

As we mentioned in introduction, we can calculate the probabilities associated to the market's position. Let $\pi_i(t) := \mathbb{P}[\theta_t = i|\mathcal{G}_t]$ for each $i \in \{-1,1\}$. Clearly, $\overline{\theta}_t = \pi_1(t) - \pi_{-1}(t) = 2\pi_1(t) - 1$. Since

$$d\overline{\theta}_t = -2\lambda\overline{\theta}_t dt + \frac{\alpha}{\sigma}(1-\overline{\theta}_t^{\,2})d\overline{W}_t,$$

a little algebra now yields

$$\pi_1(t) = p + \int_0^t \lambda\left(1-2\pi_1(s)\right)ds + \int_0^t \pi_1(s)\frac{2\alpha(1-\pi_1(s))}{\sigma}d\overline{W}_s. \tag{4.13}$$

5 The effect of intermediate announcements

In reality, there are instants at which intermediate announcements regarding the zone's net position are made. Every year, the European union aggregates submitted emission data and compares this to the quantity of allowances surrendered. The processing of emissions data for the entire zone usually takes a couple of months time, and announcements on the zone's net position are not released until mid April every year. Until that time, spot trading in the contract for delivery in the past year still takes place. As the EU so announces net results every year, we must look a bit deeper into the effects of intermediate announcements on net positions. Intuitively, one expects intermediate announcements to affect prices on more than one occasion, where the impact can be as dramatic as what happened with Dec-07 prices early May 2006. This event was also induced by intermediate announcements, then regarding the net position for the emission year 2005. The long position then implied zero value for Dec-05 allowances, but prices for Dec-07 were also affected, the difference being that its price remained positive, as risk regarding the net positions for 2007 persisted. In the remainder of this section, we will study the impact of such intermediate announceents in the framework of our model.

To be more concrete, suppose at some $t_0 < T$ the true position of the zone is revealed to the market. To ease the calculations we further assume that there will be no further announcements before time T. As it will be seen from the following calculations, this assumption does not effectively restrict the model since the formulas can be readily extended to the case with multiple announcements. Note that we still keep the incomplete information setting described in Section 4. Let, for $r \leq t$,

$$\omega_{ij}(t,r) := \mathbb{P}(\theta_t = i|\mathcal{F}_t^S, \theta_r = j), \text{ for } i,j \text{ in } E.$$

Then, it follows from Theorem 9.4 in [5] and that $\omega_{ij}(t,r) + \omega_{-ij}(t,r) = 1$ that

$$\omega_{ij}(t,r) = \delta(i,j) + \int_r^t \lambda(1 - 2\omega_{ij}(s,r))ds + \int_s^t \omega_{ij}(s,r)\frac{\alpha(i-k)(1-\omega_{ij}(s,r))}{\sigma}d\overline{W}_s,$$

where $\{k\} = E\backslash\{i\}$.

Now, we redefine $\overline{\theta}$ so that $\overline{\theta}_t = \mathbb{E}[\theta_t|\mathcal{F}_t^S, \theta_{t_0}]$ for $t > t_0$. Since θ takes values in E, we have that $\overline{\theta}_t = 2\omega_{1\theta_{t_0}}(t,t_0) - 1$, for $t > t_0$. This implies that the dynamics of $\overline{\theta}$ changes to

$$\overline{\theta}_t = \theta_{t_0} - 2\int_{t_0}^t \lambda\overline{\theta}_s ds + \frac{\alpha}{\sigma}(1 - \overline{\theta}_s{}^2)d\overline{W}_s, \tag{5.14}$$

for $t \geq t_0$. Note that for $t < t_0$ the expression for $\overline{\theta}$ is still given by (4.9). This implies that, typically, there will be a jump in $\overline{\theta}$ at time t_0 since $\overline{\theta}_{t_0-}$ will be different than θ_{t_0} as long as $0 < \mathbb{P}(\theta_t = 1) < 1$ for all t.

An attractive feature of the expression (5.14) is that it is the unique solution to the SDE defined by (4.9) but with the initial condition $\overline{\theta}_{t_0} = \theta_{t_0}$. This is of course no surprise given the Markovian structure of our model. An immediate consequence of this is that for $t \geq t_0$

$$P_t = h(t, S_t, \overline{\theta}_t) - Z_t\frac{S_t + K}{2},$$

where h is the function that is defined in Theorem 4.3, and $Z_t = \mathbf{1}_{[t=T]}(\theta_T - \overline{\theta}_T)$ as before. Now, let $\tilde{\theta}_t := \mathbb{E}[\theta_t|\mathcal{F}_t^S]$, for $t \leq t_0$. Then, for $t < t_0$

$$\begin{aligned} P_t &= \mathbb{E}^*[h(t_0, S_{t_0}, \theta_{t_0})|\mathcal{G}_t] \\ &= \mathbb{E}^*[h(t_0, S_{t_0}, \theta_{t_0}) - h(t_0, S_{t_0}, \tilde{\theta}_{t_0})|\mathcal{G}_t] + \mathbb{E}^*[h(t_0, S_{t_0}, \tilde{\theta}_{t_0})|\mathcal{G}_t] \\ &= Z_t^h + h(t, S_t, \tilde{\theta}_t), \end{aligned}$$

where

$$Z_t^h = \mathbb{E}^*[h(t_0, S_{t_0}, \theta_{t_0}) - h(t_0, S_{t_0}, \tilde{\theta}_{t_0})|\mathcal{G}_t], \tag{5.15}$$

for $t \leq t_0$[1] Summing up the above calculations we have the following theorem.

[1]Note that $\mathbb{E}^*$ is the expectation operator under the minimal martingale measure in this new setting, which will typically be different than the minimal martingale measure $\mathbb{P}^*$ from the previous section.

Theorem 5.1 *Suppose that the true state of θ is revealed at time $t_0 < T$. The fair price of EUA0 contracts is given by*

$$P_t = \begin{cases} h(t, S_t, \overline{\theta}_t) - Z_t \frac{S_t + K}{2}, & \text{for } t \geq t_0, \\ Z_t^h + h(t, S_t, \tilde{\theta}_t), & \text{for } t < t_0, \end{cases}$$

where h is the function defined in Theorem 4.3 and Z^h is as defined in (5.15). P has a jump at t_0 and the jump size equals

$$\Delta P_{t_0} = h(t_0, S_{t_0}, \theta_{t_0}) - h(t_0, S_{t_0}, \tilde{\theta}_{t_0}) - Z_{t_0}^h.$$

Proof. The equation for P follows from the definition of the fair price of EUA0 contracts. In order to see the second assertion, first note that $\mathcal{G}_t = \mathcal{F}_t^S$ for $t < T$. Since all $\mathcal{F}^S$-martingales are continuous, it follows that $(Z_t^h)_{0 \leq t \leq t_0}$ is a continuous $\mathcal{F}^S$-martingale. Thus,

$$P_{t_0-} = h(t_0, S_{t_0}, \tilde{\theta}_{t_0}) + Z_{t_0}^h,$$

since h and $\overline{\theta^0}$ are continuous. ∎

Notice that $\mathbb{E}^*[\Delta P_{t_0} | \mathcal{G}_t] = 0$ for all $t \leq t_0$. However, ΔP_{t_0} is typically nonzero unless θ_{t_0} is deterministic.

6 Numerical study

In this section, we will will investigate the stylized facts of our model as apparent from a simulation run. We will pick a random path of EUA1 price process based on the dynamics in (2.1), after which we can calculate the fair price process for the EUA0 contract given the evolution of EUA1 prices by numerically solving the boundary value problem in Theorem (4.3). The fact that the EUA0 price is essentially expressed in terms of the conditional expectation in Definition (4.1) provides us with a means to avoid the resolution of the complex PDE. It implies that calculation of the carbon spot price can also be calculated in a Monte Carlo routine, which is technically simpler and less prone to issues of numerical instability of grid solutions.

For this purpose we first need to simulate a path for the process θ modelling the net position of the market. The parameters used in order to produce the following graphs are given in the table below.

Parameter	Value
α	-0.5
σ	1
λ	2
K	40
μ	0.4
S_0	20.00
θ_0	-1
p	0.5
T	240

We set $T = 240$ and divide this period into 240 trading dates of equal length. Figure 2 shows the simulated path for the position θ (gray) of the zone. We next proceed to simulate a path for EUA1 prices from equation (2.1). Note that we have compensated the value chosen for $\alpha = -0.5$ by taking $\mu = 0.4$. This is important, as the drift can otherwise become very large, resulting in EUA1 prices running out of range. The resulting simulation for EUA1 prices is plotted as the green curve in Figure 3. The next task is to estimate $\bar{\theta}$ out of the filtering equations in (4.11) and (4.12). To commence this, we start from the starter value $\bar{\theta}(0) = 2p-1$, here equal to zero. At every time t, we compute $\bar{\theta}$ as a conditional expectation across 1000 Monte Carlo steps, simulating paths post time-t and averaging over the payoff for EUA0 prices at maturity.

We opted for a scenario where the true state of the market is revealed after 192 time steps, or 4/5 of the interval. This event is marked by the plots for θ and $\bar{\theta}$ in Figure 2 coinciding as $\bar{\theta}$ 'jumps' onto $\theta = +1$, highlighting an apparent long position for the market as soon as the intermediate announcement is released. Figure 2 shows the path for θ (green) versus that for $\bar{\theta}$ (black) for the parameter choices at hand. After the 'jump' in the $\bar{\theta}$ path, the market perception of the true state reverts back towards zero. This is due to the observed increase in EUA1 prices. Note that due to the negative relationship with the sign of the net position and the drift of the EUA1 prices, the observed upward drift in EUA1 prices will lead the market to think that the net position might be changing from long to short. On the other hand , the terminal value for θ equals $+1$ after 240 time steps, implying that the market eventually ended up long in this simulation. As a result, we observe prices for EUA0 (black) collapse to zero at the end of the trading period, as shown in Figure 3.

The effect of the intermediate announcement of an apparent long position after 192 steps is that EUA0 prices strongly decrease compared to the EUA1 *driving force* behind them at that point in time. The paths for both EUA0 and EUA1 prices are shown in Figure 3. The intermediate announcement of a long position after 192 time steps is marked by a sharp decline in EUA0 prices, compared to the prevailing price levels for EUA0. Beyond this point, EUA1 prices rally, and the EUA0 contract partly mimics this behaviour due to the change in the market's perception of the unobserved net position as explained in the last paragraph. However, at maturity, the zone ends up long, implying EUA0 prices dropping to zero after 240 steps.

The relationship between EUA0 and EUA1 prices from the simulation performed is remarkably similar to the historical evolution of the Dec-07 and Dec-08 prices in Figure 1, where an intermediate announcement in April 2006 (after about 220 time steps) regarding the (long) ETS position led to a dramatic collapse in carbon prices. The resemblance could be improved by extending the set of admissible states for θ from just two elements. Recall that the slow but steady decline of DEC07 prices to level 0 was largely due to the fact that the information release in April 2006 revealed that NAP levels were so generous that there is almost no possibility for the zone to end up short at the end of the year. Obviously, it is not possible to measure how long or short the zone is by a θ process with only two states. Therefore, a better fit to the data could be more easily achived by introducing more states to the model. However, we still want to limit ourselves to a model with two states which

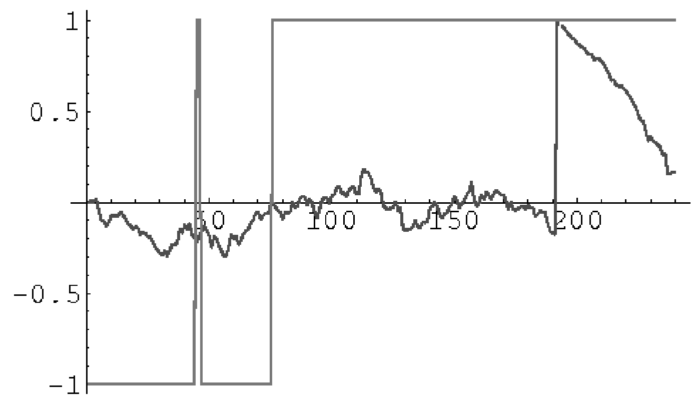

Figure 2: True θ (gray) and corresponding $\bar{\theta}$ path (black) for a long market scenario

is good enough to highlight the ability of our model to capture the main features of carbon prices, and in order to stay consistent with the simple exposition of the theory outlined in the previous sections. Moreover, it is also worth to mention that as the carbon market is still not mature the price behaviour is not always caused by rational actions in an efficient financial market, thereby making the capture of certain price movements impossible by a consistent and rigorous model.

7 Concluding remarks and extensions

We discussed the pricing and hedging of EUA contracts traded within the EU ETS scheme when banking of permits are not allowed. The key idea of our model is to make the drift component of EUA1s dependent on the net position of the market at large. This combined with the no-arbitrage relation between EUA0 gave us the setting where we can infer the long probabilities from EUA1 prices and calculate arbitrage free prices for EUA0 contracts. Since the setting is incomplete by nature there exists an interval for arbitrage-free prices and we have chosen one price based on a local risk minimising criteria. As seen in Section 3 one can come up with explicit formulas for pricing and hedging under the assumption that the market's net position is common knowledge among the market participants. Under the more realistic setting where the market does not observe the net position directly the price of EUA0 contracts and associated hedging portfolios can be obtained by solving a boundary value problem. The analytical solution for the PDE cannot be obtained, however, a numerical solution for the price via Monte Carlo simulation can be computed easily and fast.

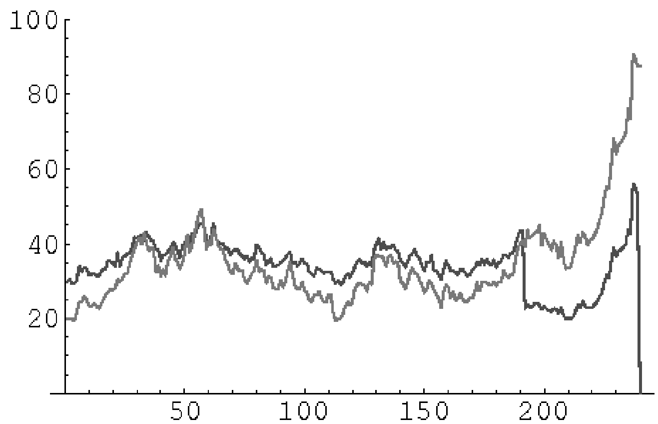

Figure 3: Simulated EUA price history for a long market scenario

According to the latest EU Directive [9] banking of carbon permits is allowed during and after the second phase of Kyoto protocol. Our approach can be modified to incorporate this optionality of the current carbon trading as well. In order to price correctly the EUA0 contracts, we need to first determine what the appropriate condition relating the spot and forward contracts at time T should be. When the banking is in effect, the unused permits will no longer be worthless after the year in which they were issued since they can be used later to cover the the excess emissions. This implies that although the identity $S_T + K = P_T$ holds when $\theta_T = -1$, $P_T > 0$ when $\theta_T = +1$. This will make the pricing of EUA0 contracts complicated since now the pricing of such contracts will depend on the utility functions of the agents holding EUA0 contracts and the probability of them needing these permits at a future date. Indeed they will have to determine a price at which they will be indifferent between selling it now to other agents who will need these contracts to cover their positions or keeping these permits for possible future use, either by themselves or others. In order to simplify the matters one may *assume* that there exists a representative agent representing the overall traders and this agent is risk neutral, i.e. has an objective to maximise expected profit. Let's also suppose that there are only 2 periods to trade, one being $[0, T]$ and other $[T, 2T]$. This is only to ease exposition and the extension to a multiperiod setting will be straightforward. Thus, the indifference price of EUA0 contracts for the representative agent is given by

$$P_T = \begin{cases} S_T + K, & \text{if } \theta_T = -1, \\ \mathbb{E}[\mathbf{1}_{[\theta_{2T}=-1]} S_{2T} | S_T, \theta_T = 1] & \text{if } \theta_T = 1. \end{cases}$$

In order to find $\mathbb{E}[\mathbf{1}_{[\theta_{2T}=-1]} S_{2T} | S_T, \theta_T]$ we need to find the joint distribution of (θ_{2T}, S_{2T}) given (θ_T, S_T). However, this joint distribution can be easily found using the well-known Kolmogorov's equations given the Markov property of (θ, S). Once $\mathbb{E}[\mathbf{1}_{[\theta_{2T}=-1]} S_{2T} | S_T, \theta_T]$

the method explained in earlier sections can now be utilised to calculate price process of EUA0 contracts.

Another possible improvement for the model would be to allow nonsymmetric changes in drift, in the sense that news regarding short positions reduce the drift more strongly than continued conviction on the overdimensioning of NAP volumes. Note that when θ takes only two values the linearity assumption is without loss of generality once we assume that the drift is only a function of θ. Thus, in order to include such non-symmetric changes in drift one should model θ by a Markov chain with more than two states. This would as well give chances for a better calibration of the model using the market data.

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A Glossary

EU ETS The European Union Emission Trading Scheme (EU ETS) is the largest multinational emissions trading scheme in the world. The ETS covers over 12.000 installations in energy and industrial sectors, representing nearly half of the EU carbon dioxide (CO_2) emissions. Operators of installations covered by the ETS within the EU must monitor and annually report on their CO_2 emissions and are obliged to return an amount of emission allowances sufficient to cover their annual emissions.

First & second phase In the beginning of each phase allowances are issued for multi-year periods at once. The first ETS phase extended from 2005 until 2007 and the second phase from 2008 until 2012.

NAP Within each of the phases, operators obtain a number of free allowances under the terms of the National Allocation Plan (NAP). The extent in which these free allowances cover the actual needs of the operator varies across the participant industrial sectors. Any remaining allowances needed must be covered in the open market where EU CO_2 allowances (EUA's) are traded.

EUA The European Union Allowance (EUA) for CO_2 is the main vehicle enabling operators still short allowances after NAP issuance to cover their actual carbon dioxide emissions. Contracts entailing the delivery of EUA at various future instants are traded under the ETS. The maturity for a standardized contract is December. For instance, the contract called 'Dec-08' delivers the allowance for one tonne of CO_2 into the buyers' ETS account in December 2008. Trading and price formation of these allowances happens Over The Counter (OTC) or via an exchange such as the European Climate Exchange (ECX).

EUA0 & EUA1 In our paper, EUA0 represents the price of the EUA contract for the allowance delivering in the current year. The contract delivering an allowance in the subsequent year is denoted by EUA1. Examples treated in this paper are the case where EUA0 refers to the Dec-07 contract delivering in the first phase, while EUA1 depicts the Dec-08 contract delivering in the second phase (no banking from one year to another is permitted in this case).